Overview of Special Relativity

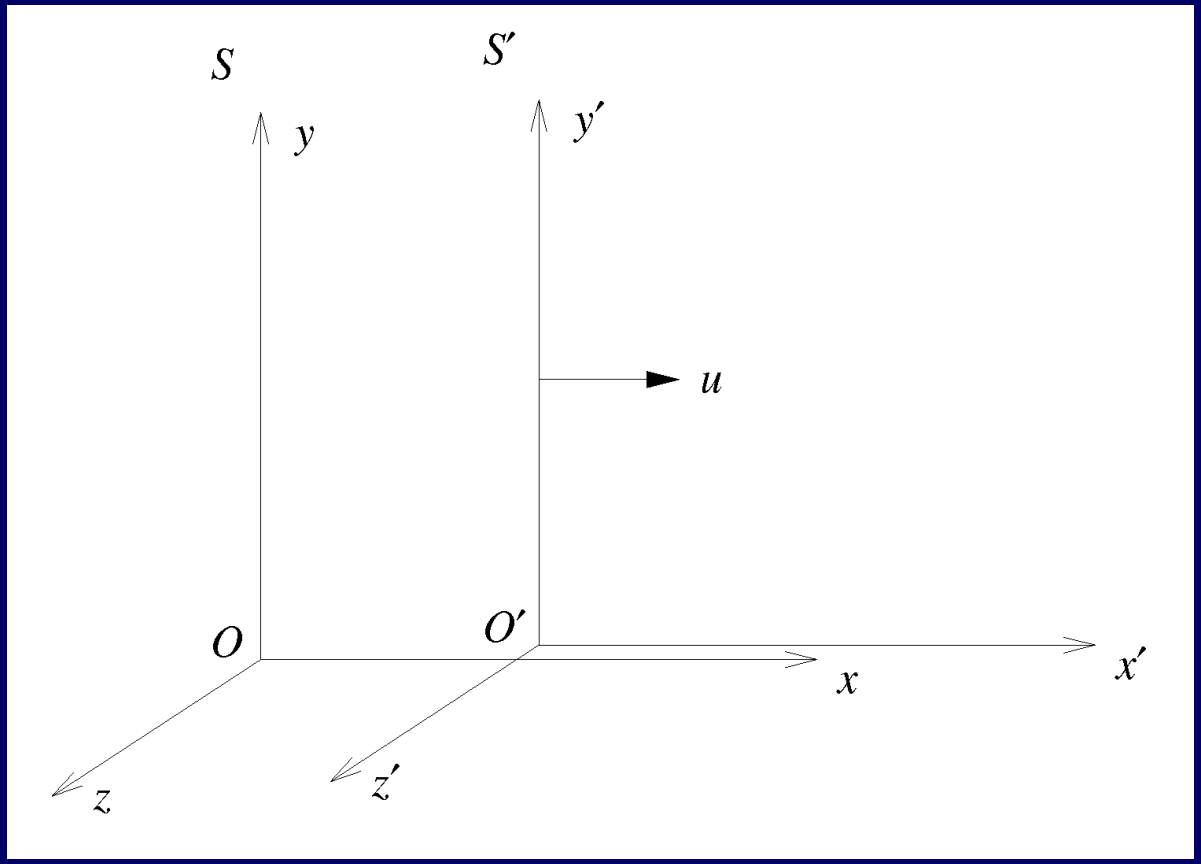
The Bare Minimum

History

- Galilean transformation equations (pre-relativity, incorrect)
  - The only thing you can measure is the relative velocity, not the absolute velocities of each frame.

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

  - $v_x' = v_x - u$
  - $v_y' = v_y$
  - $a' = a$

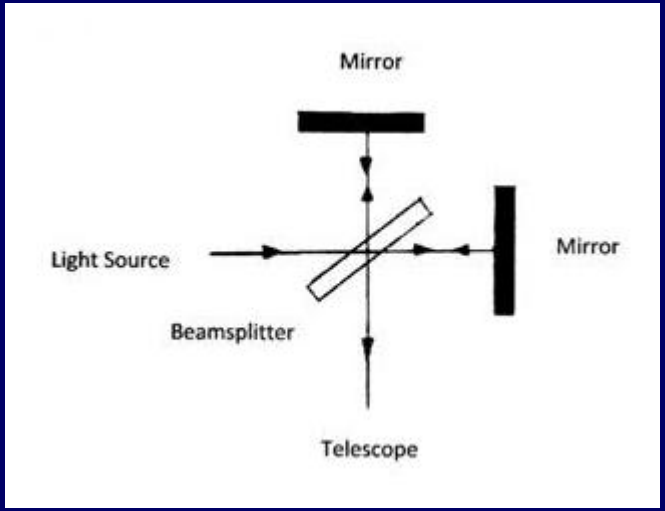


History

- Maxwell's Equations:

  $c = 2.99792458 \times 10^8$ m/s

- Does light move relative to a "luminiferous ether"!?
  - Earth orbits and spins through ether
    - Speed of light slower in direction of orbital motion?
  - Michelson-Morley Experiment, 1887
    - Null result
- Galilean transformations cannot be correct
  - Newton's Laws are in jeopardy



Einstein's Postulates

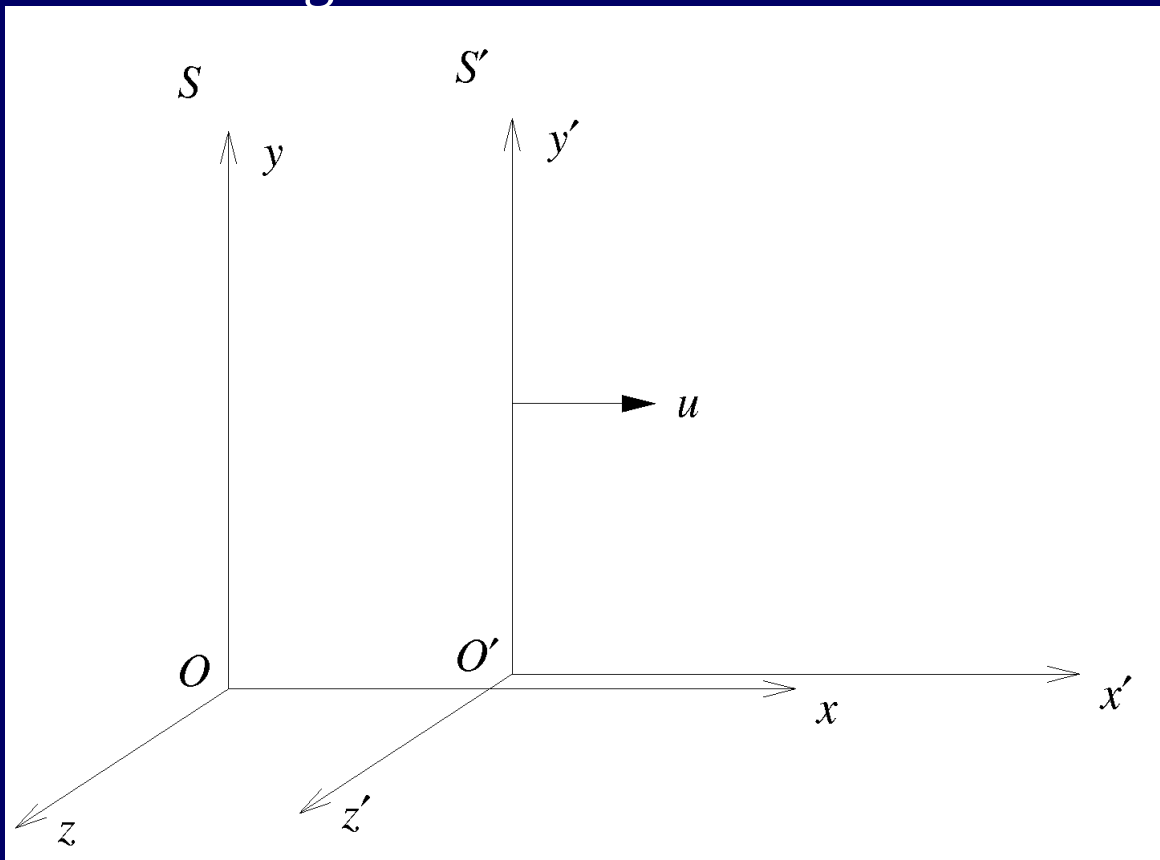
- **The Principle of Relativity**
  - The laws of physics are the same in all inertial reference frames
- **The Constancy of the Speed of Light**
  - Light travels through a vacuum at a constant speed *c* that is independent of the motion of the light source
- Lorentz Transformations

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$



Time Dilation and Length Contraction

- Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \geq 1$$

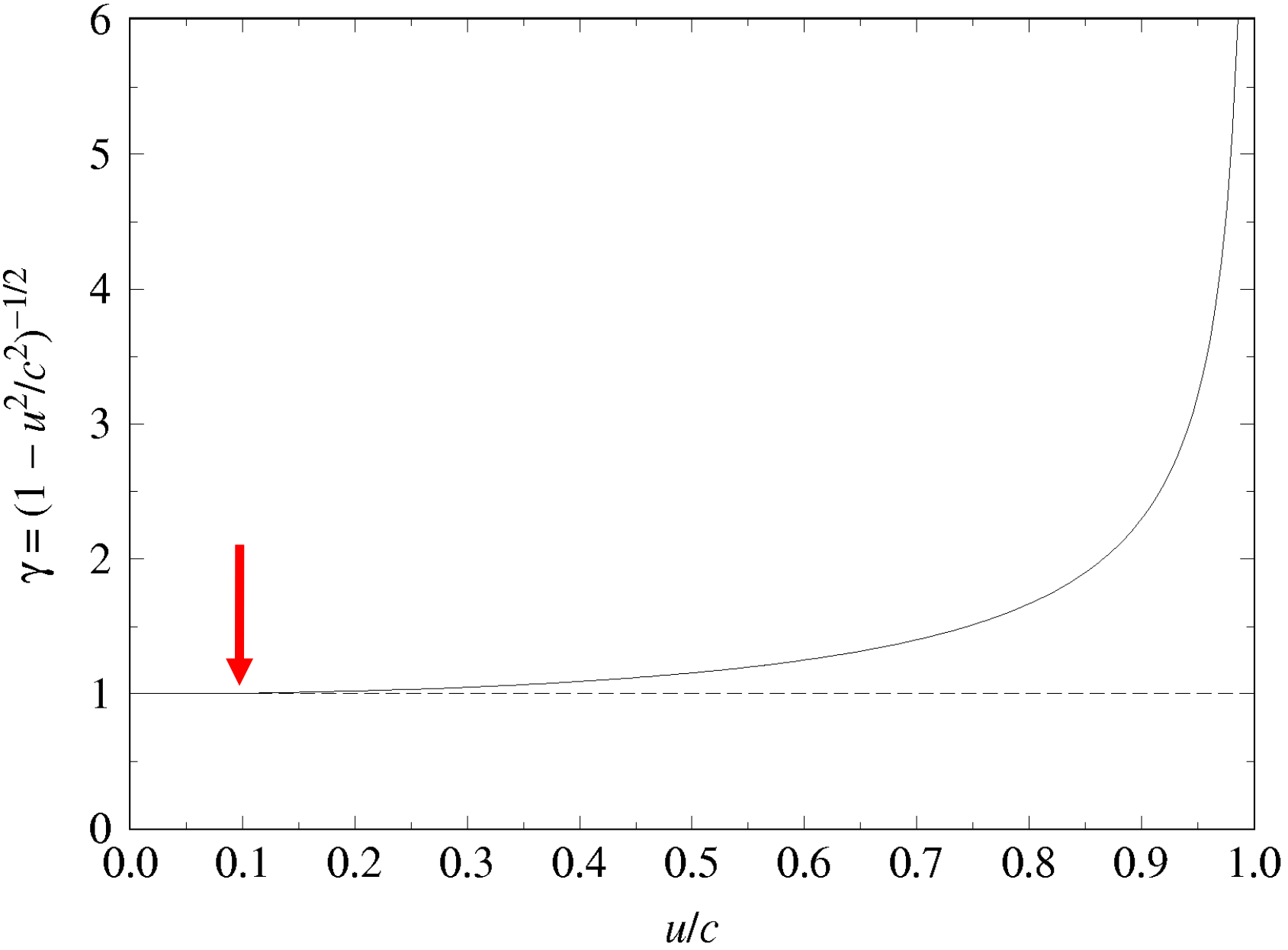
- Time Dilation: moving clocks run slower (to the observer in the “rest” frame)

$$\Delta t_{moving} = \frac{\Delta t_{rest}}{\sqrt{1 - u^2/c^2}} = \gamma \, \Delta t_{rest}$$

- Length contraction: moving objects appear shorter

$$L_{moving} = L_{rest} \sqrt{1 - u^2/c^2} = L_{rest} / \gamma$$

Lorentz Factor

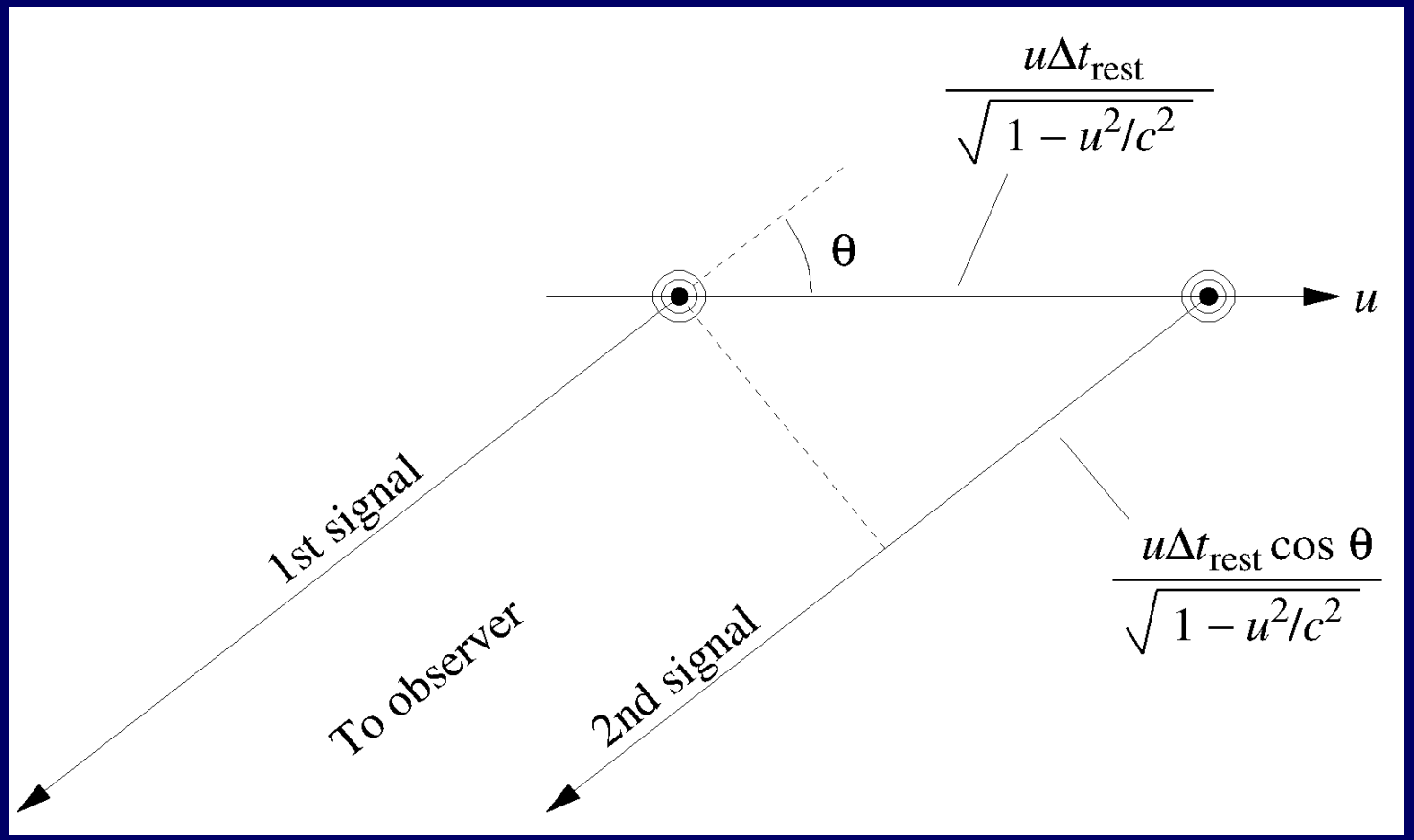


Relativistic Doppler Shift

Different from sound waves: $\nu_{obs} = \nu_{rest}\ ((c+vo)/(c-vs))$

- There is a *transverse* Doppler effect

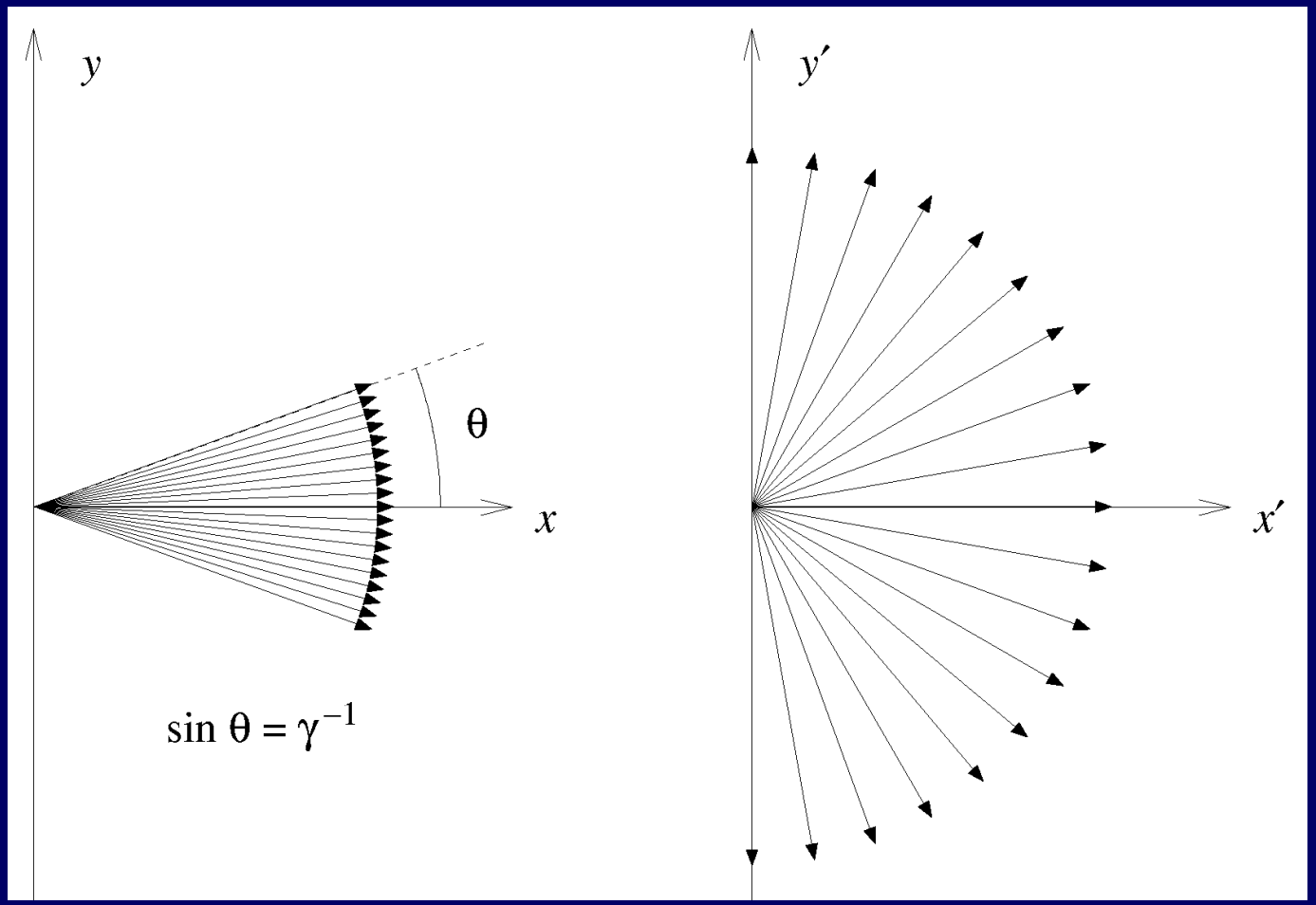
$$v_{obs} = \frac{v_{rest}\sqrt{1-u^2/c^2}}{1+(u/c)\cos\theta} = \frac{v_{rest}\sqrt{1-u^2/c^2}}{1+v_r/c}$$



Relativistic Headlight Effect

- Light emitted isotropically into the forward hemisphere is concentrated into a narrow cone
  - From velocity transformation equations, however $v=v'=c$

$$\sin\theta=\gamma^{-1}$$



Relativistic Momentum and Energy

- Momentum $\vec{p} = \dfrac{m\vec{v}}{\sqrt{1 - v^2/c^2}} = \gamma m \vec{v}$

- Kinetic Energy $K = mc^2(\gamma - 1)$

- Energy $E = \gamma mc^2$

  $E_{rest} = mc^2$

  – A lot of people choose to say $E = mc^2$

    m_o is the rest mass $m = \gamma m_o$

Invariants

- Total energy of particle with rest mass m and momentum p.
- $$E^2 = p^2 c^2 + m^2 c^4$$
- The total energy of a closed system of particles is invariant.