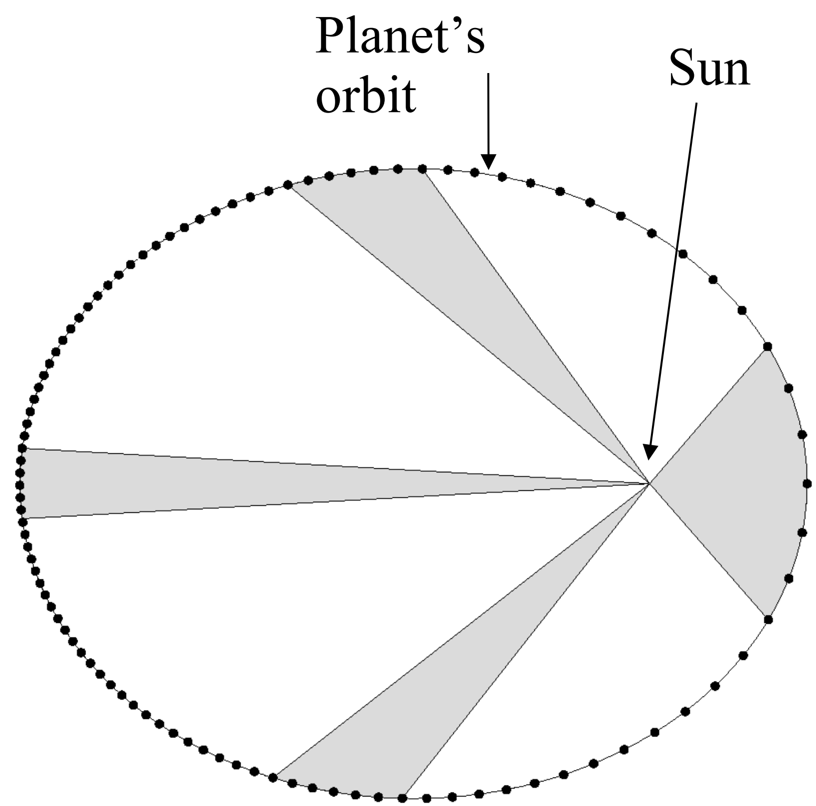
Kepler’s Laws of Motion

- 1609 in Astronomia Nova (The New Astronomy)

- First Law – A planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse.
- Second Law – A line connecting a planet to the Sun sweeps out equal areas in equal time intervals
  - Several areas associated with the time interval of “six” are shown
    - They all have equal areas



Kepler's Third Law of Motion

From *Harmonia Mundi* (1619) (Harmony of the Worlds)

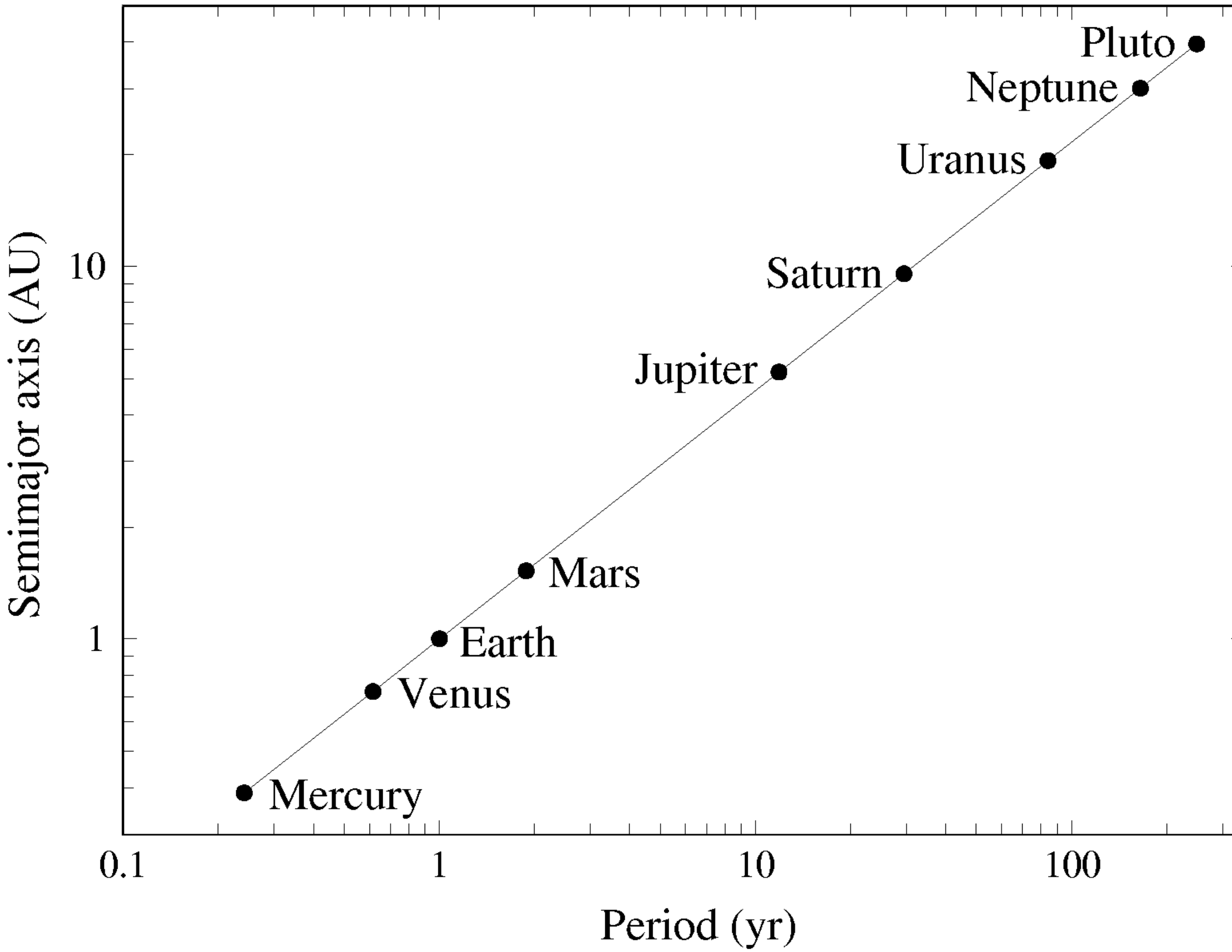
$$\boxed{P^2 = a^3}$$

P = orbital period

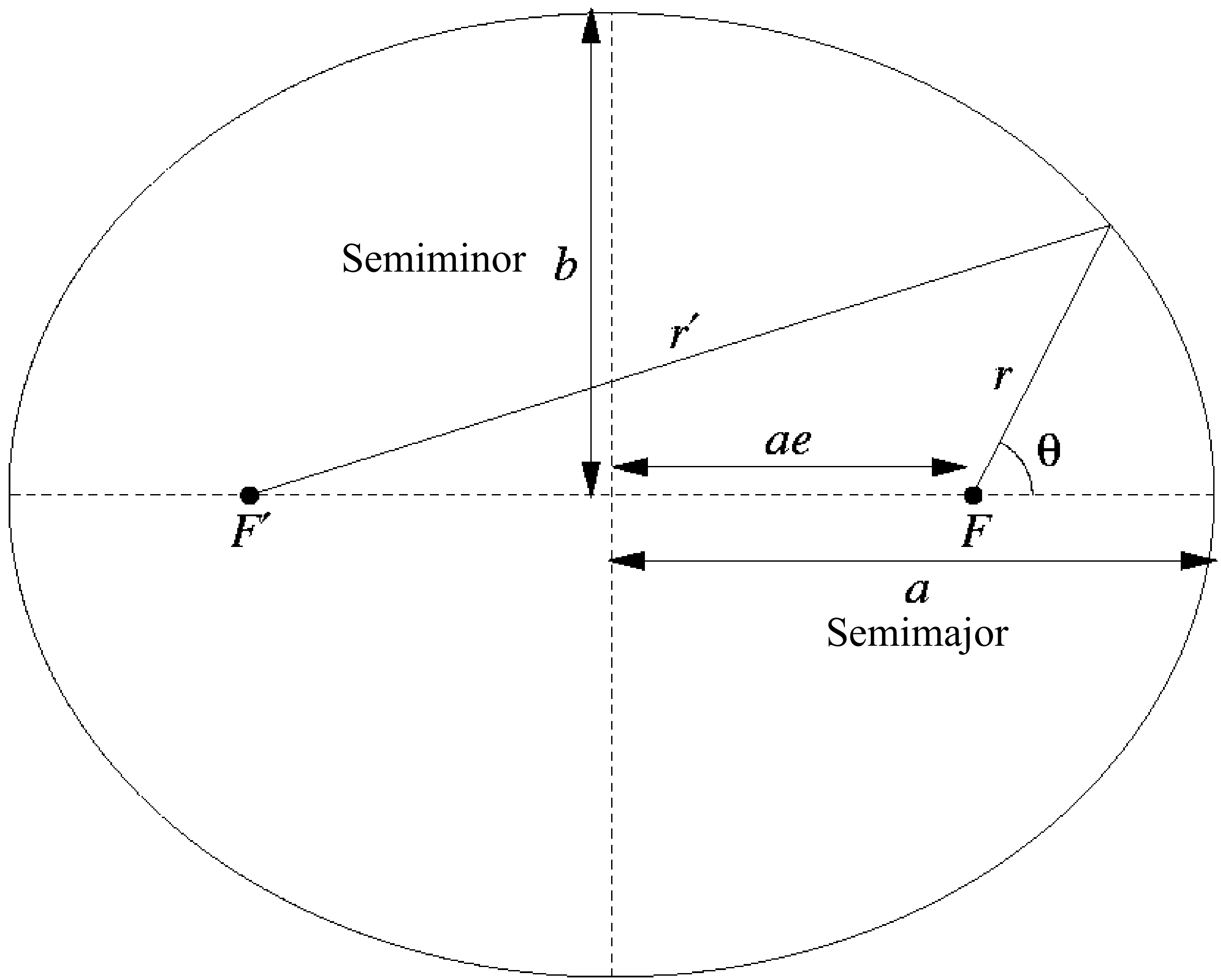
a = semimajor axis

"Power law" slope is 2/3:

$$\log(P^2) = \log(a^3)$$
$$2\log(P) = 3\log(a)$$
$$\log(a) = \frac{2}{3}\log(P)$$

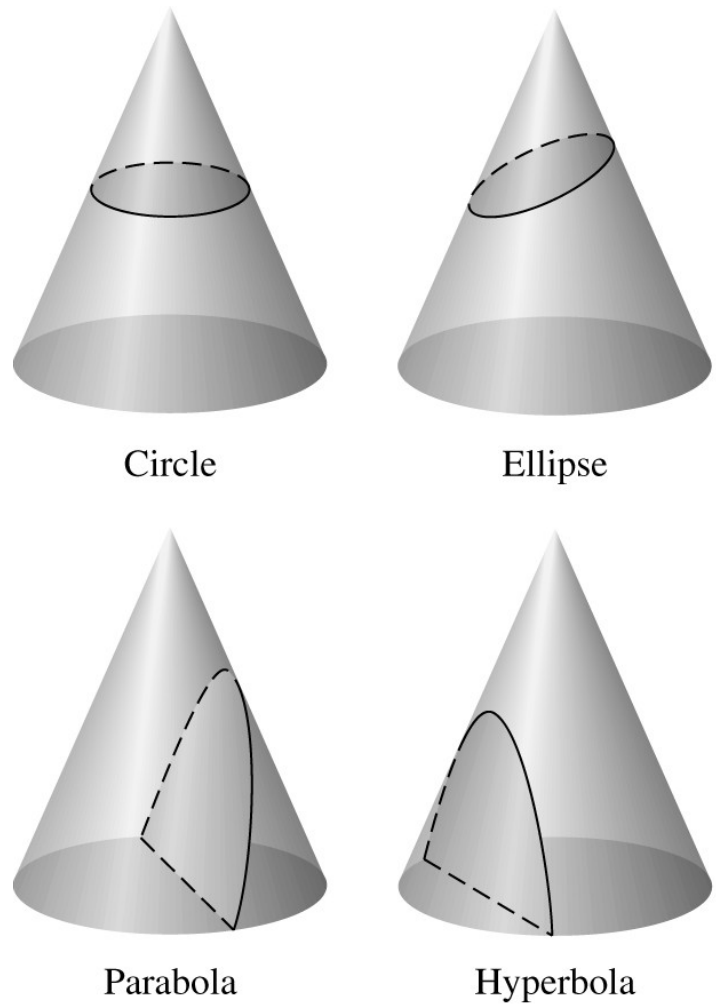


Ellipses



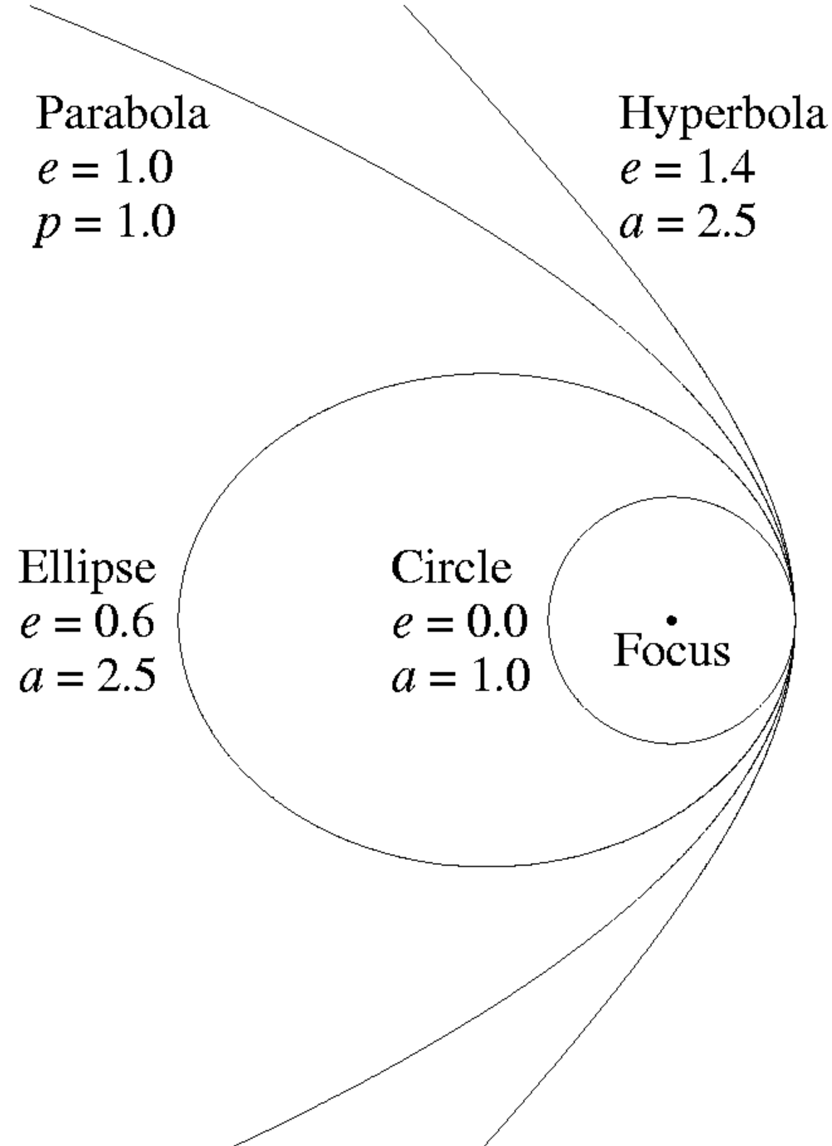
Conic Sections

- Intersection of a plane with a cone
- Parabola – plane is parallel to a side
- Hyperbola – plane is parallel to central axis
- All are possible orbits (elliptical orbits most common)



Conic Sections

- All are possible in celestial mechanics.
- “p” is closest approach for parabolic orbit

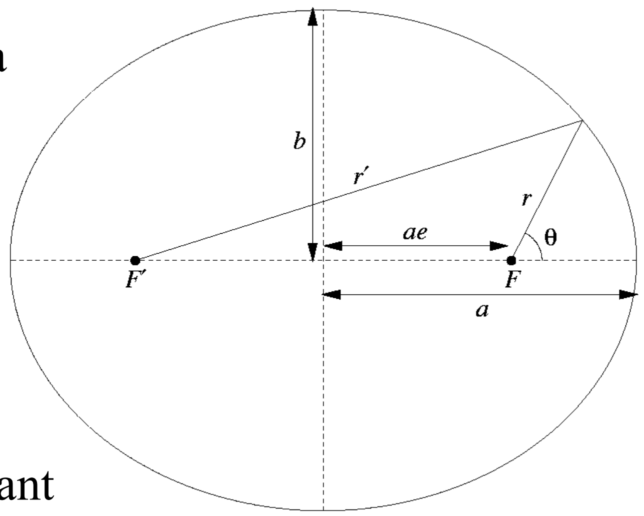


$r = a$	$e = 0$	Circle
$r = \dfrac{a\left(1-e^2\right)}{1+e\cos\theta}$	$0 \le e < 1$	ellipse
$r = \dfrac{2p}{1+\cos\theta}$	$e = 1$	parabola
$r = \dfrac{a\left(e^2-1\right)}{1+e\cos\theta}$	$e > 1$	hyperbola

$$r = \frac{L}{1+e\cos\theta}$$

General, where L = dist from Focus to curve along line perp to major axis (semi-latus rectum)

Ellipse Drawing

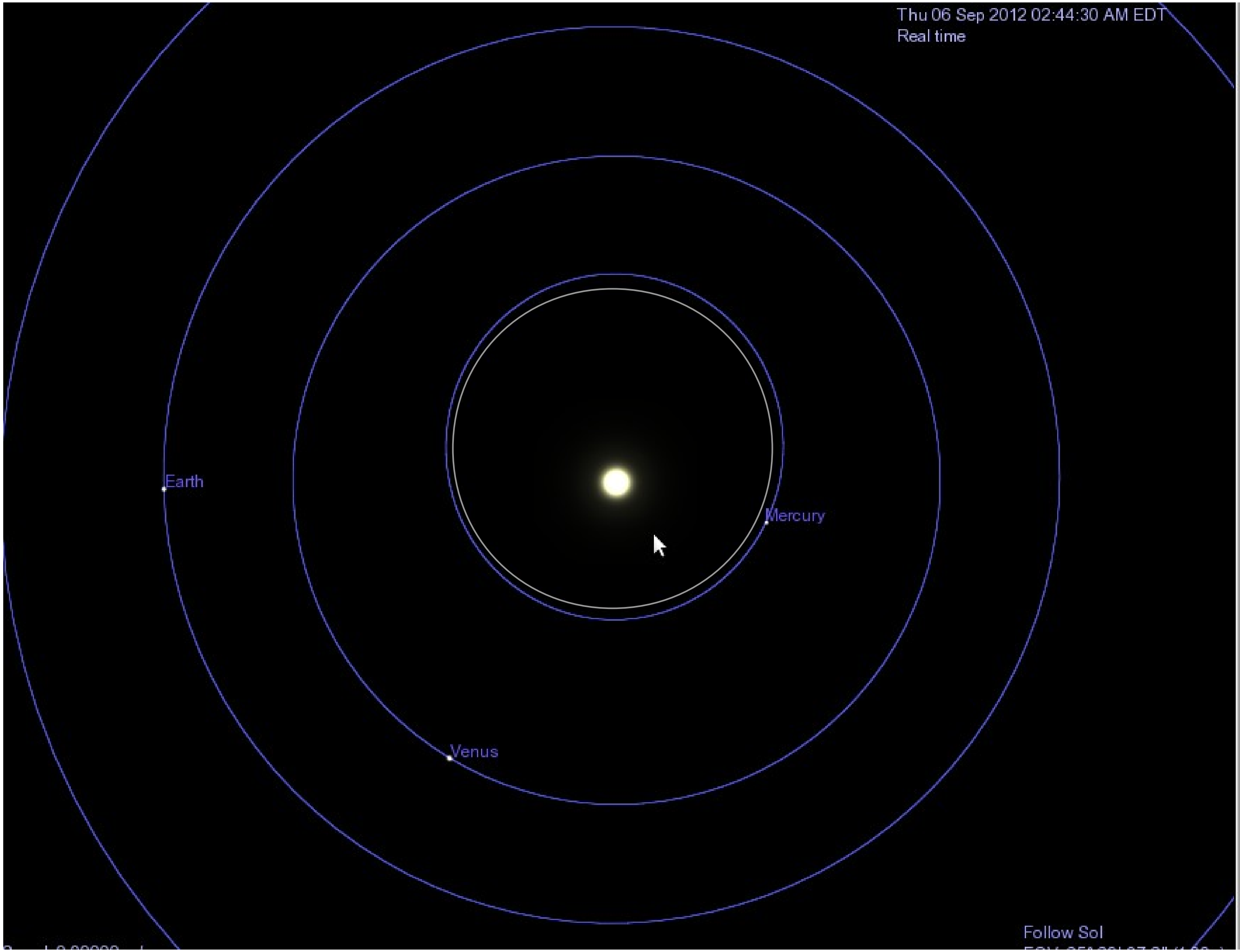


After drawing your ellipse on graph paper by keeping a pencil snug against a string looped loosely around two tacks, do the following:

1) Mark center “O”.
2) Mark F and F' (foci).
3) Measure and label a and b (in mm).
4) Measure and label ae.
5) Draw point (labelled “P”) on ellipse in the 1st quadrant position. Draw, label, and measure r and r'. Measure θ.
6) Confirm r + r' = 2a
7) Calculate eccentricity using e = ae/a
8) Calculate eccentricity using $e=\sqrt{1-\left(\frac{b}{a}\right)^2}$
9) Confirm that $r=a(1-e^2)/(1+e\cos\theta)$
10) Measure x and y for P, where (x,y)=(0,0) at center (not focus)
11) Confirm the Cartesian coordinate equation for the ellipse using point P: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

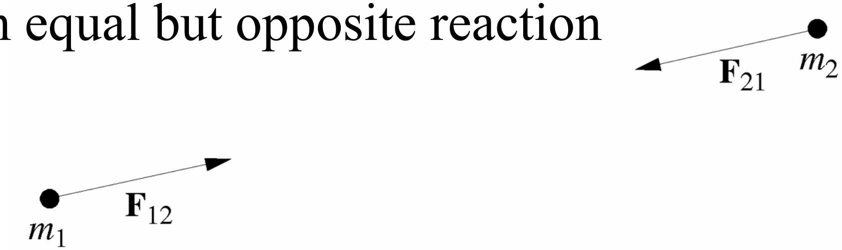
Ellipses – actual orbits

(September 2012)



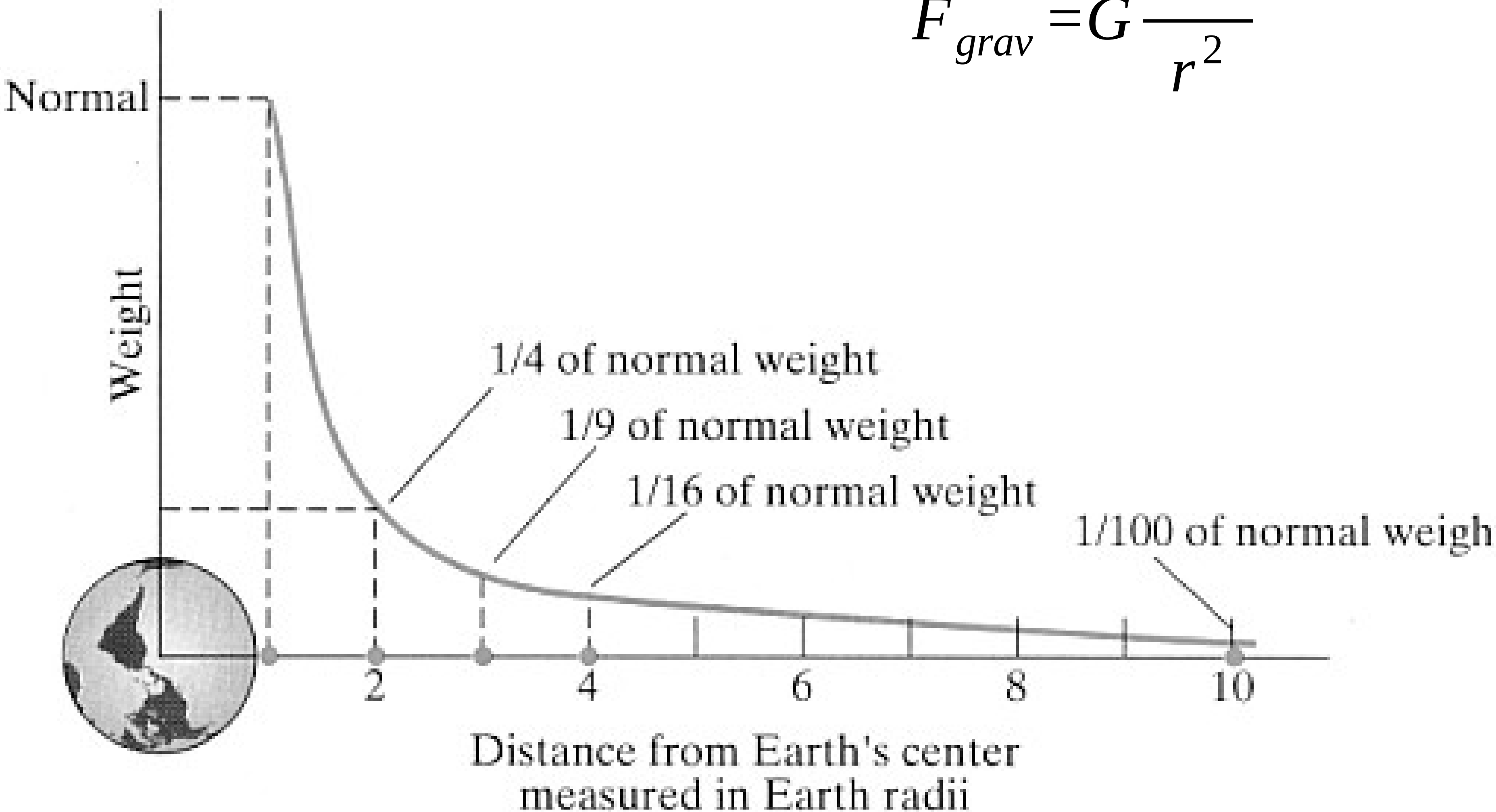
Newton's Laws of Motion

- 1^{st} Law – Law of inertia
  - An object at rest remains at rest and an object in uniform motion remains in uniform motion unless acted upon by an unbalanced force.
  - An ***inertial reference frame*** is needed for 1^{st} law to be valid
  - A non-inertial reference frame is accelerating (e.g. In car going around a curve you feel a fictitious force)
- 2^{nd} Law – $\mathbf{a} = \mathbf{F}_{\mathbf{net}}/m$ or $\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$
  - The net force (sum of all forces) acting on an object is proportional to the object's mass and its resultant acceleration.
  - Inertial mass, m, does not appear to be different from gravitational mass
- 3^{rd} Law
  - For every action there is an equal but opposite reaction



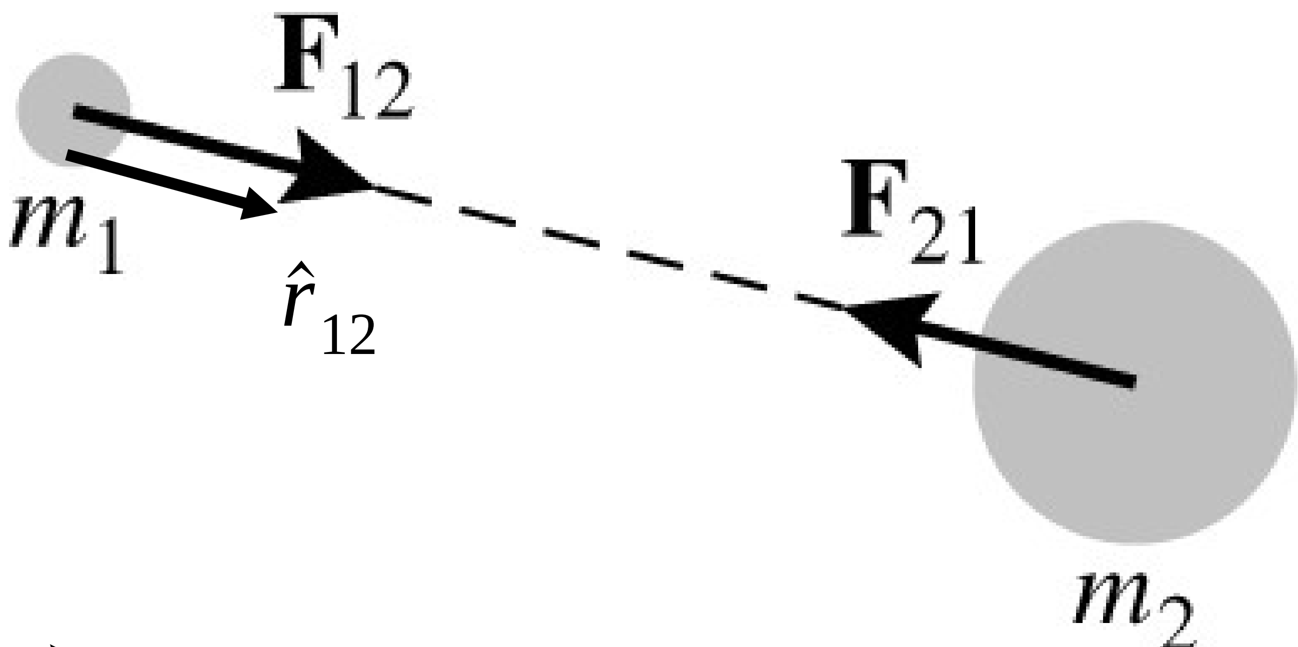
Universal Law of Gravitation

$$F_{grav} = G\frac{Mm}{r^2}$$



Universal Law of Gravitation

$$\vec{F}_{12} = G\,\frac{Mm}{r^2}\,\hat{r}_{12}$$

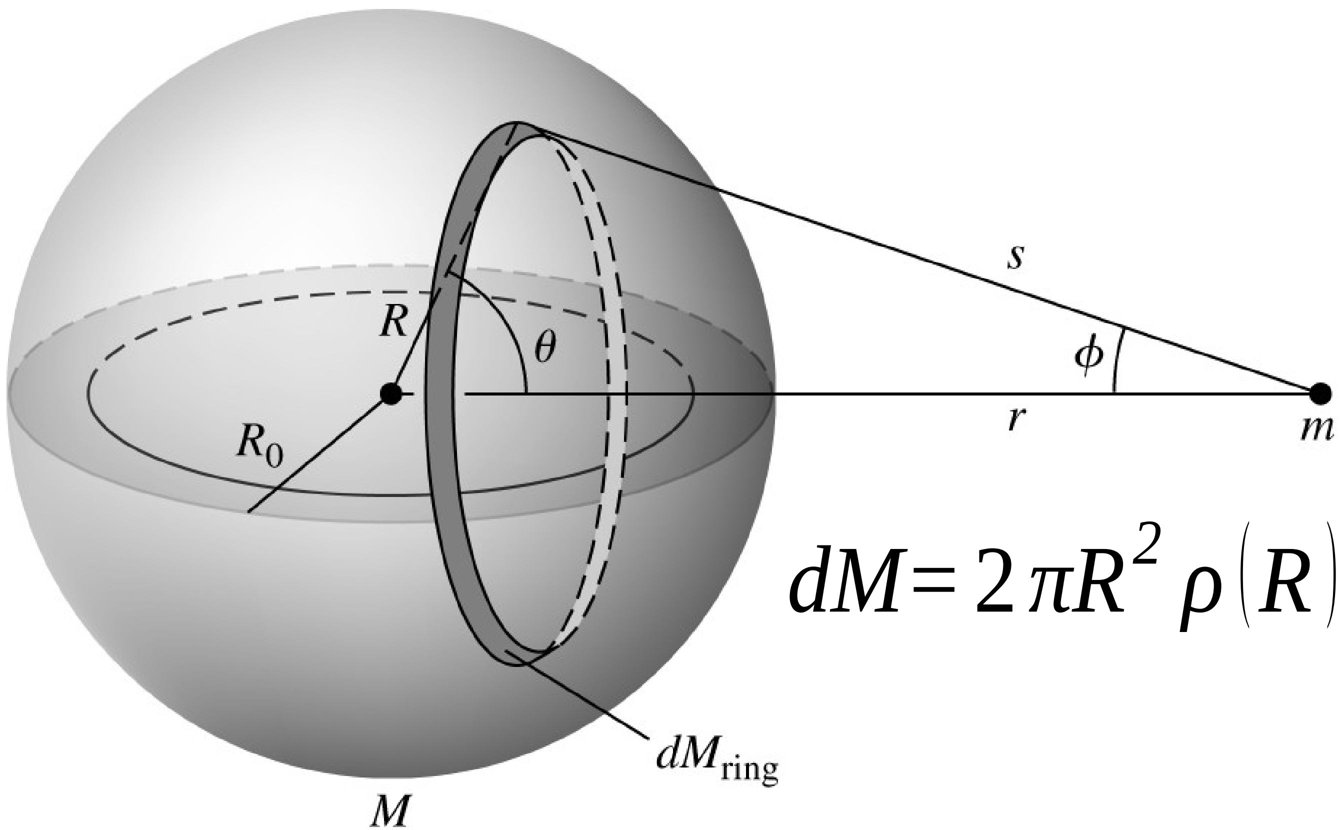


$\vec{F}_{12}$ is force <u>on 1 by 2</u>.
Unfortunately, this is opposite the convention used in PHYS 2321 (Coulomb's Law)

Shell theorems for gravity:

) The Force on *m* due to a uniform shell of mass is the same as the force due to a point mass at the center of the shell with the same total mass as the shell.

) The force of gravity inside of a uniform shell is zero.



$$dM = 2\pi R^2 \rho\left(R\right) \sin\theta dR d\theta$$

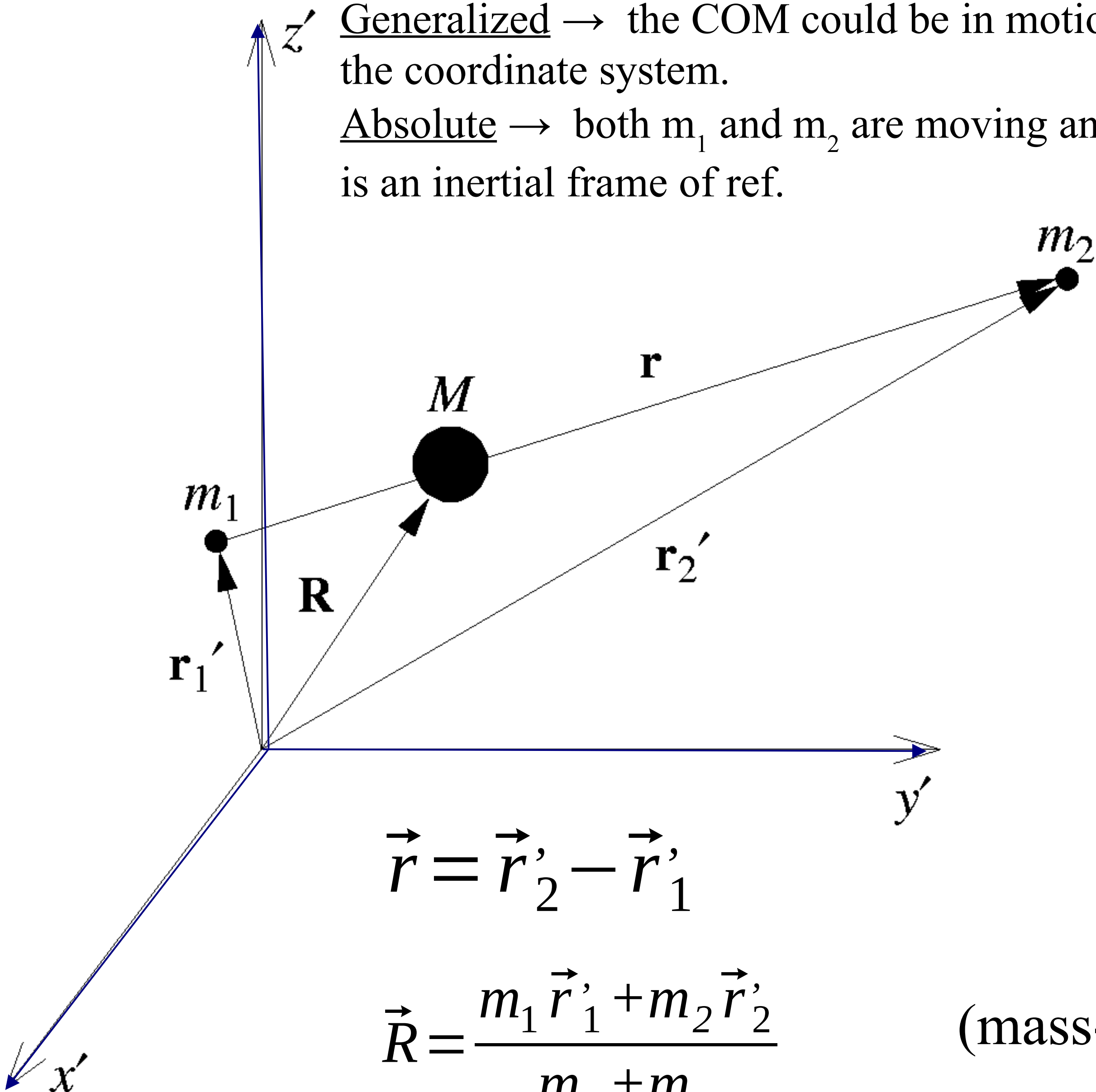
(See Ch. 2 derivation of $F_{shell} = GM_{shell}m/r^2$.)

Binary Orbits

Generalized, absolute coordinates.

Generalized → the COM could be in motion relative to the coordinate system.

Absolute → both m_1 and m_2 are moving and the coord sys is an inertial frame of ref.



$$\vec{r} = \vec{r}'_2 - \vec{r}'_1$$

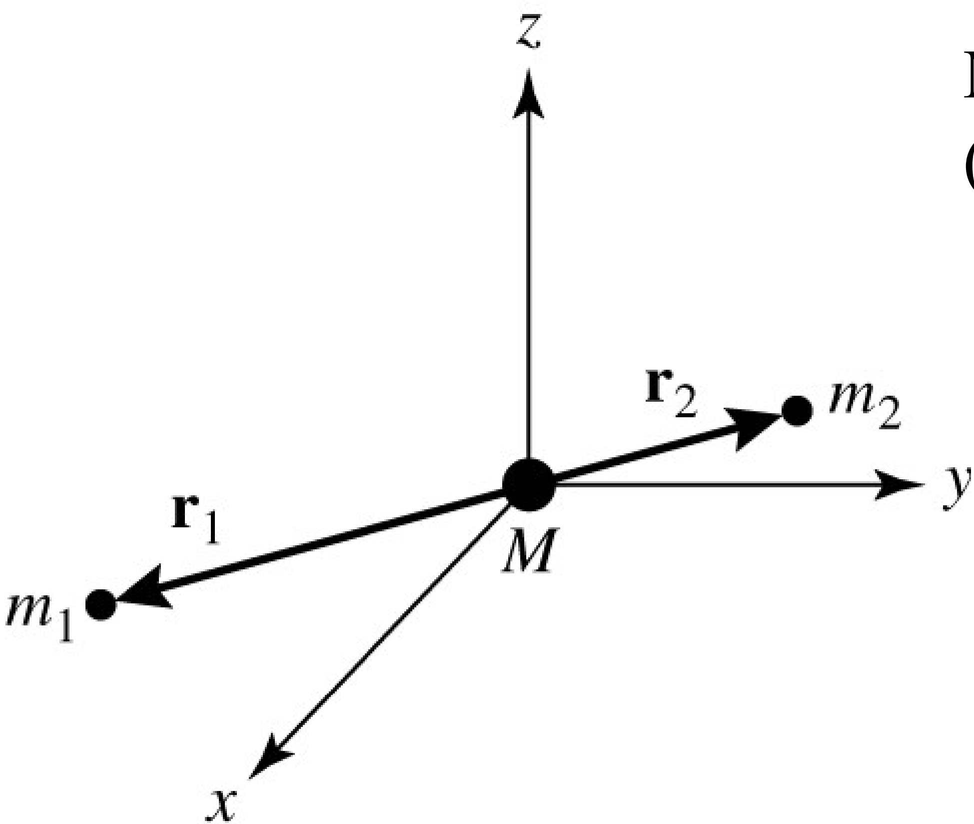
$$\vec{R} = \frac{m_1 \vec{r}'_1 + m_2 \vec{r}'_2}{m_1 + m_2}$$

(mass-wtd average)

Binary Orbits

Absolute coordinates.

Absolute → both m_1 and m_2 are moving and the coord sys is an inertial frame of ref.
The COM is placed at the origin. It is labeled with the total mass $\mathbf{M = m_1+m_2}$.



M is closer to the bigger mass (here m_2).

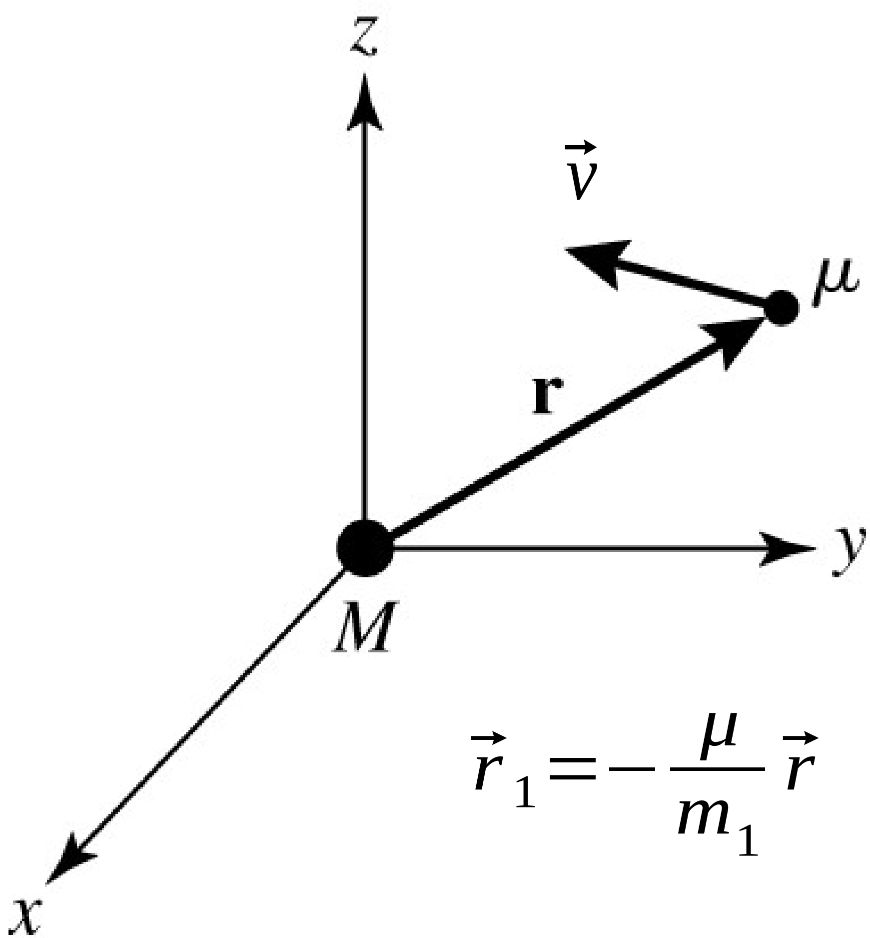
$$m_1 r_1 = m_2 r_2 \text{ so}$$

$$r_1/r_2 = m_2/m_1$$

Binary Orbits

Relative coordinates.

Relative → shows orbit of moving, *reduced mass* μ around a stationary *total mass* M.



$$\vec{r} = \vec{r}'_2 - \vec{r}'_1$$

$$|\vec{r}| = r_1 + r_2$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1 m_2}{M}$$

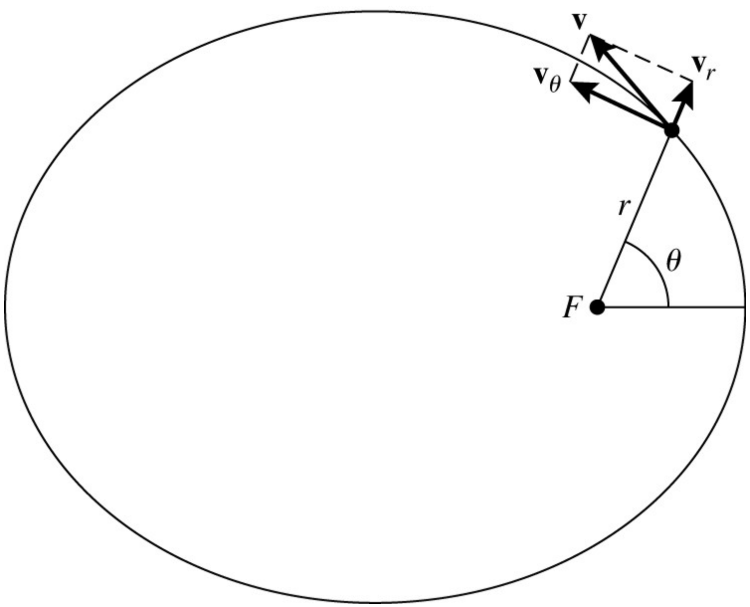
$$\vec{r}_1 = -\frac{\mu}{m_1}\vec{r} \qquad \vec{r}_2 = \frac{\mu}{m_2}\vec{r}$$

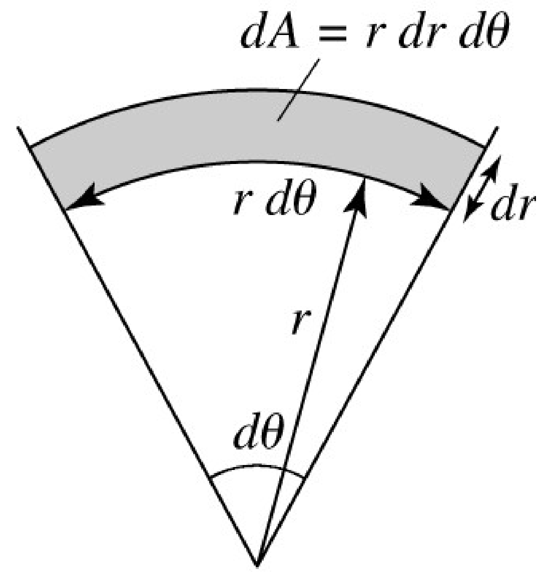
Since $\mathbf{v} = d\mathbf{r}/dt$, $\mathbf{v}_1 = -\mu/m_1\,\mathbf{v}$, and $\mathbf{v}_2 = \mu/m_2\,\mathbf{v}$

Binary Orbits

Absolute coordinates and velocity.

Velocity vector is only purely tangential at perihelion and aphelion.





$$v_\theta^2 + v_r^2 = v^2$$

$$v_\theta = r\frac{d\theta}{dt}$$

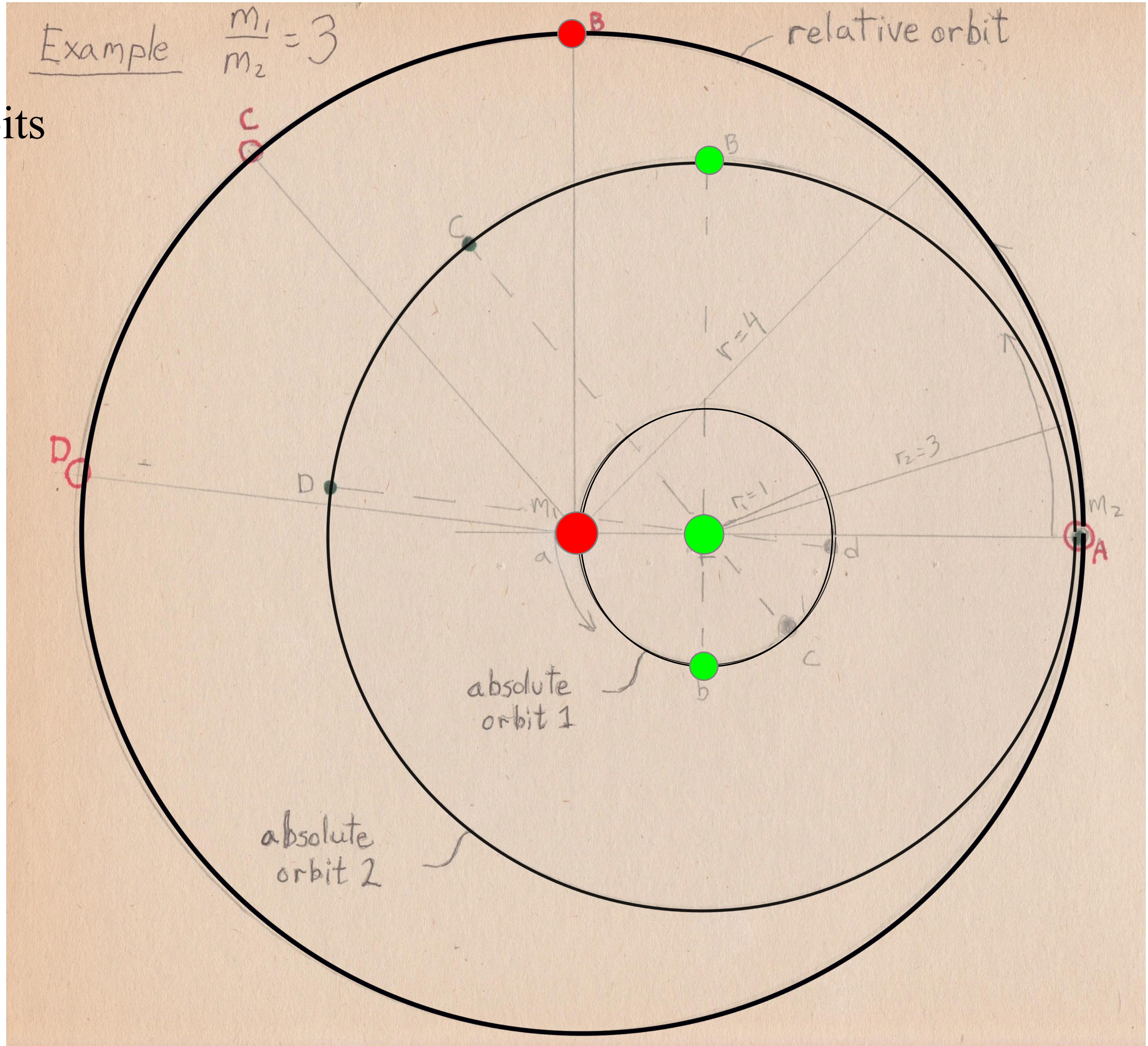
$$v_r = \frac{dr}{dt}$$

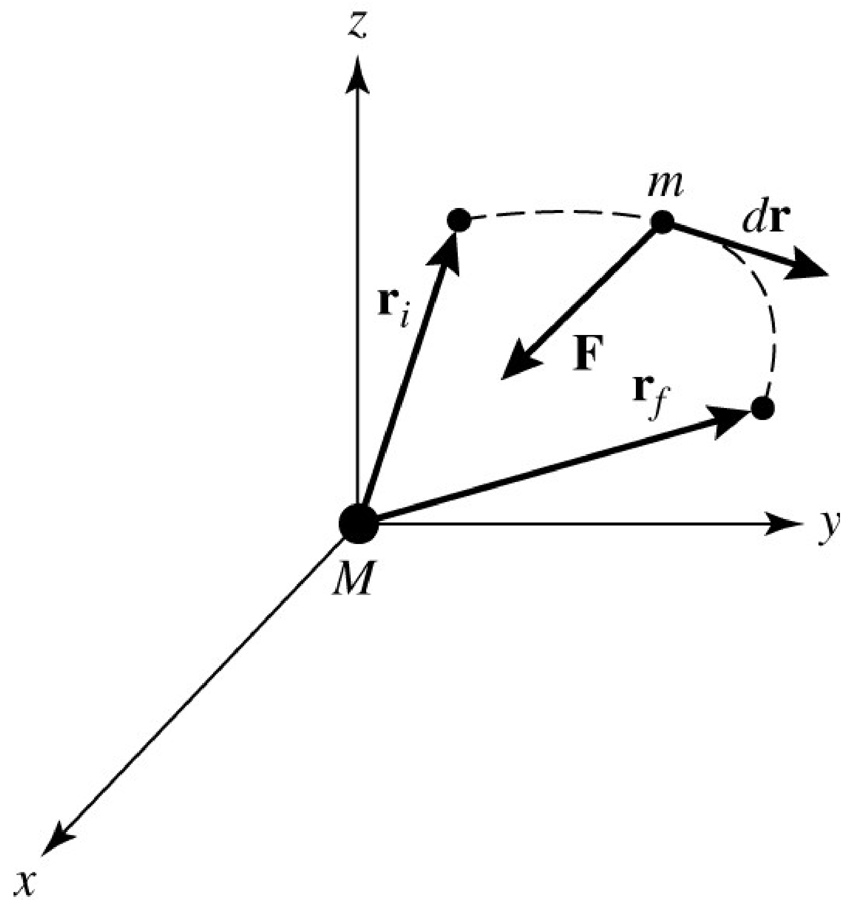
$$dA_{sector} = \int_0^R r dr d\theta$$

$$dA_{sector} = \left[\frac{R^2}{2} - 0\right] d\theta$$

$$dA_{sector} = \frac{R^2}{2} d\theta$$

Binary Orbits





Work by gravity depends on direction of net force vector relative to the direction of motion.