Classical Wave Description of Light

James Clerk Maxwell

- Related charges, currents, E-fields, and Mag-fields
- Unified Electricity and Magnetism into four equations
  - Electric Fields from charges
  - Electromagnetic Induction
    - Electric fields from changing magnetic fields
  - No magnetic monopoles (N/S pairs)
  - Electromagnet
    - Magnetic fields from moving charges (currents)

- Predicted traveling waves in Electric and Magnetic Fields (E-M waves) when charges accelerate or currents change
- No medium needed – happens in “empty space”
  - Ether theory (medium for E-M waves) has been disproved
    - the Greeks 5th element

Maxwell's Equations

- Gauss's Law

$$\nabla\cdot\vec{E}=\frac{1}{\varepsilon_o}\rho$$

- Faraday's Law

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

- No magnetic monopoles

$$\nabla\cdot\vec{B}=0$$

- Ampere's law with Maxwell's correction

$$\nabla\times\vec{B}=\mu_o\vec{J}+-\mu_o\varepsilon_o\frac{\partial\vec{E}}{\partial t}$$

E & M Waves

- Classical wave equation

$$\nabla^2 f = \frac{1}{v^2{}_o} \frac{\partial^2 f}{\partial t^2}$$

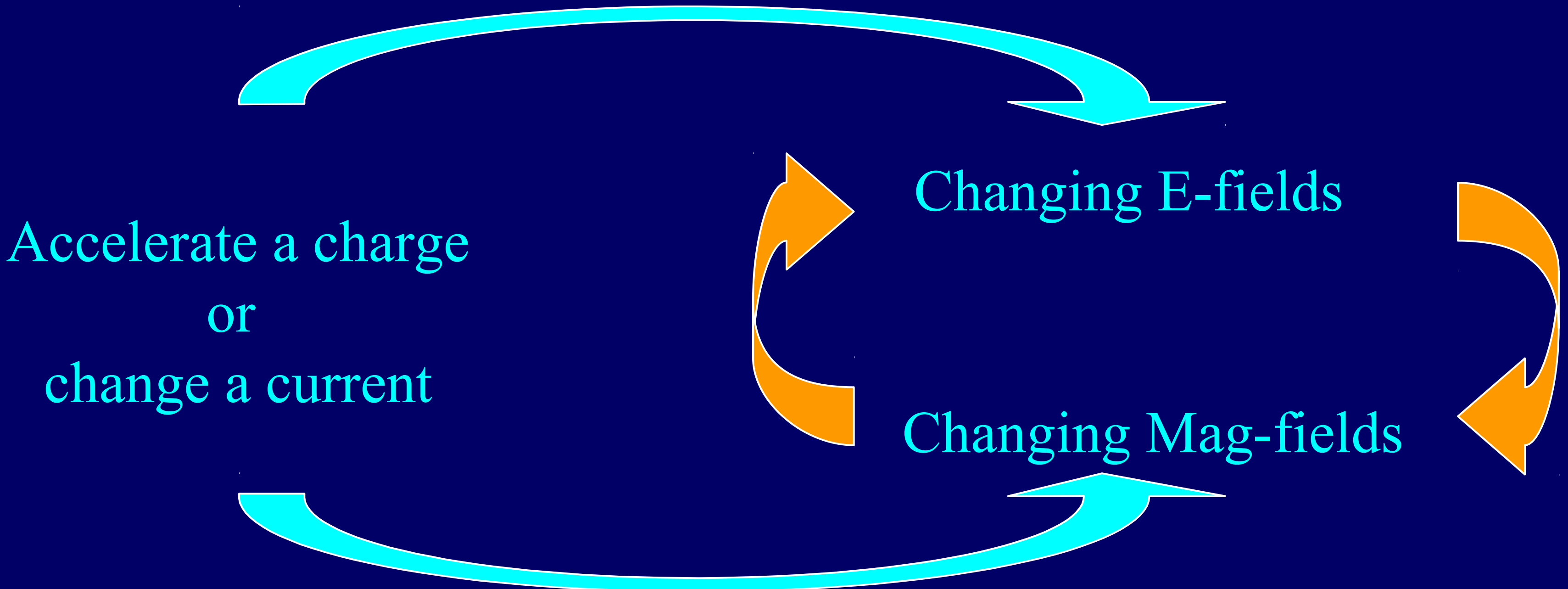
- From Maxwell's Eqs

$$\nabla^2 \vec{E} = \mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_o \varepsilon_o \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 2.99792458 \times 10^8 \, ms$$

E-M Waves



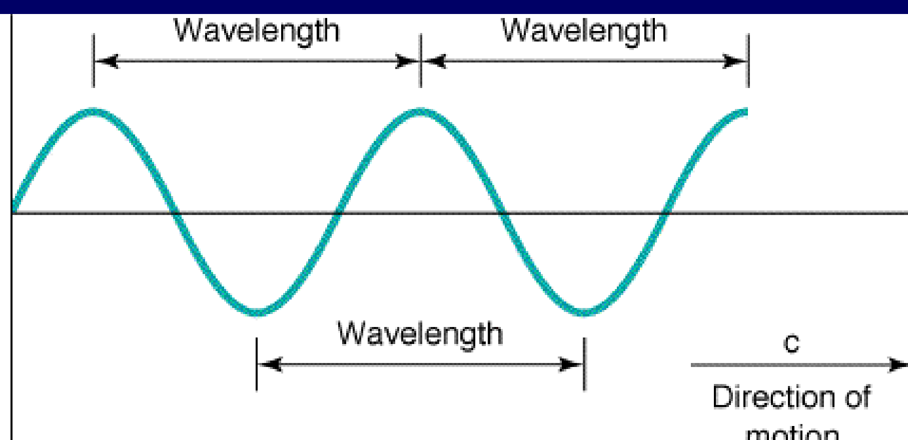
Self-propagating E-M wave

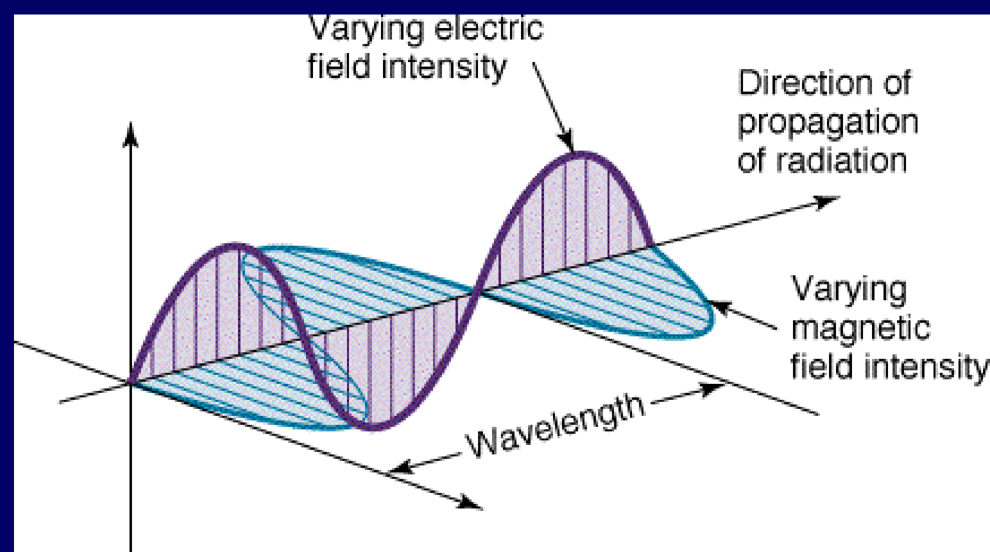
Wave Nature of Light

- Some Properties:
  - No medium needed (very strange)

    speed = c = 299792458×10^{8} m/s
  - Transverse wave
    - Polarization
      - The E-field interacts in matter (electrons)

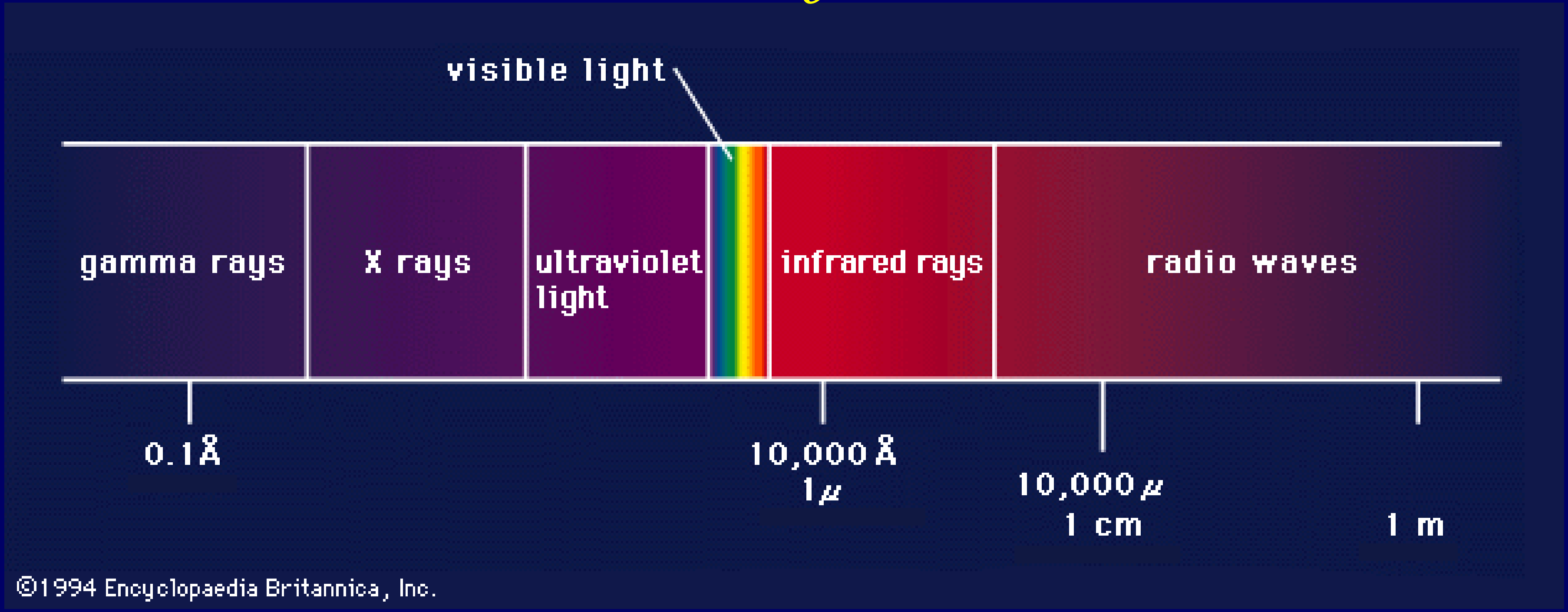
$$c = \lambda f$$





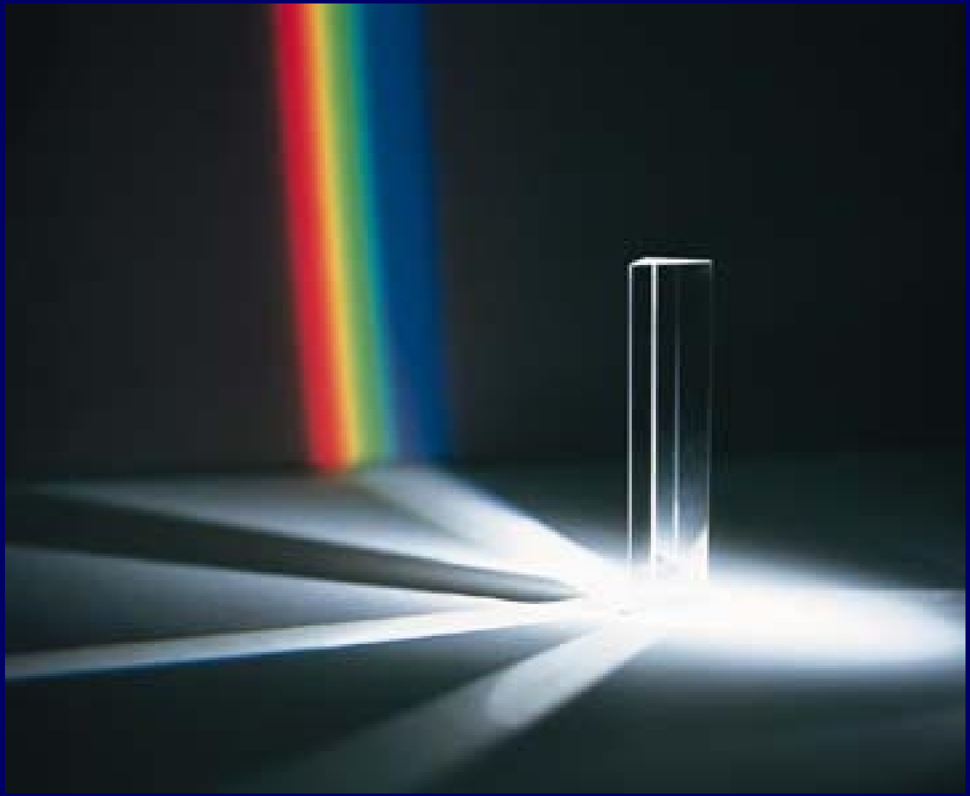
Wave Nature of Light

$$c=\lambda f$$



The Visible Spectrum

- White Light
  - mixture of all colors
  - ROY G. BIV
  - long λ to short λ
  - low *f* to high *f*
  - low Energy to high



$$c = \lambda f$$

$0.7 \times 10^{-6} - 0.4 \times 10^{-6}\, m$ or $700 - 400\, nm$

$$1\, nm = 1 \times 10^{-9}\, m$$

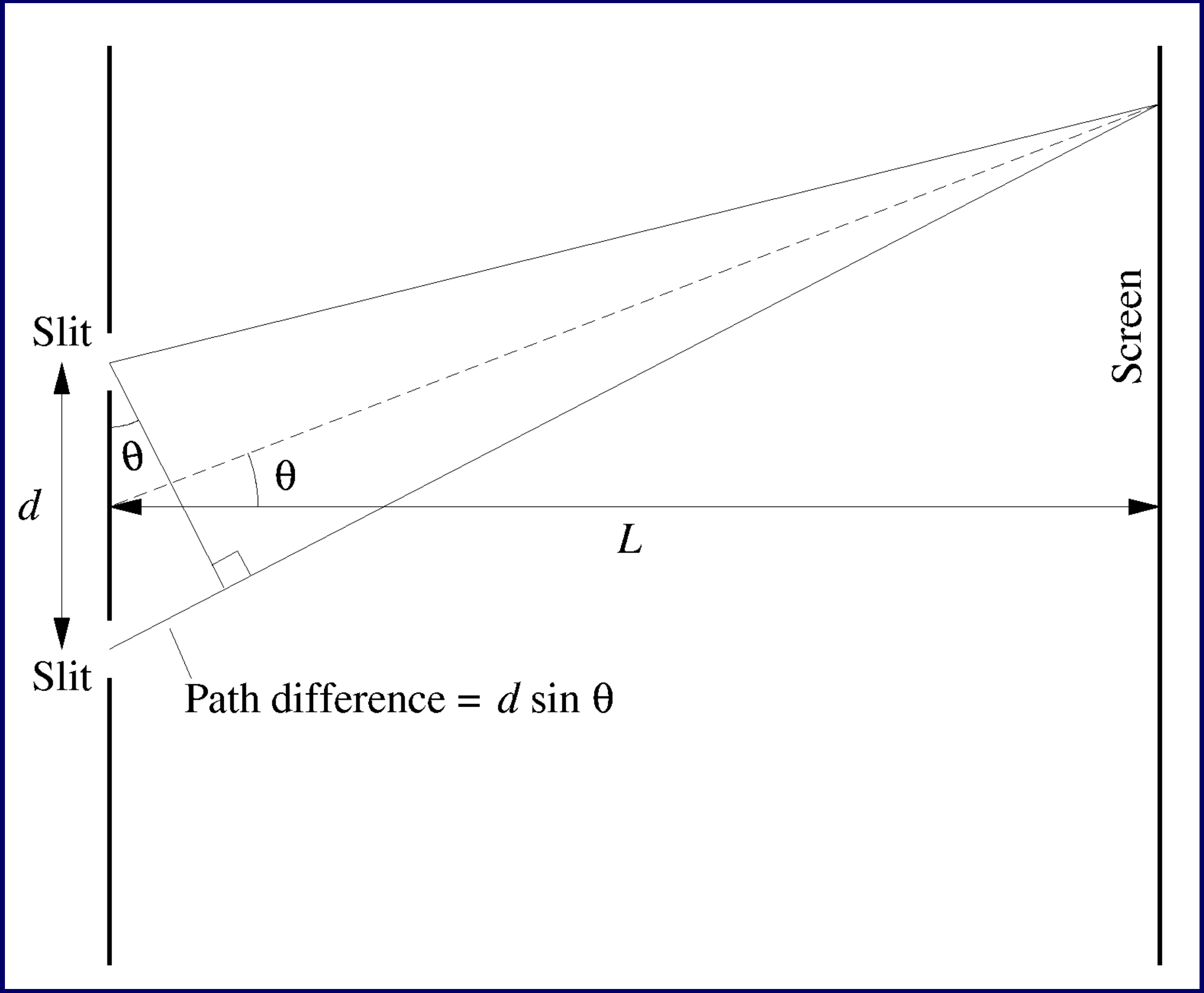
Evidence of waves:

- Interference or Principle of Superposition
  - Young’s Double Slit Experiment

Young's Double Slit Experiment

Two narrow slits –> diffraction

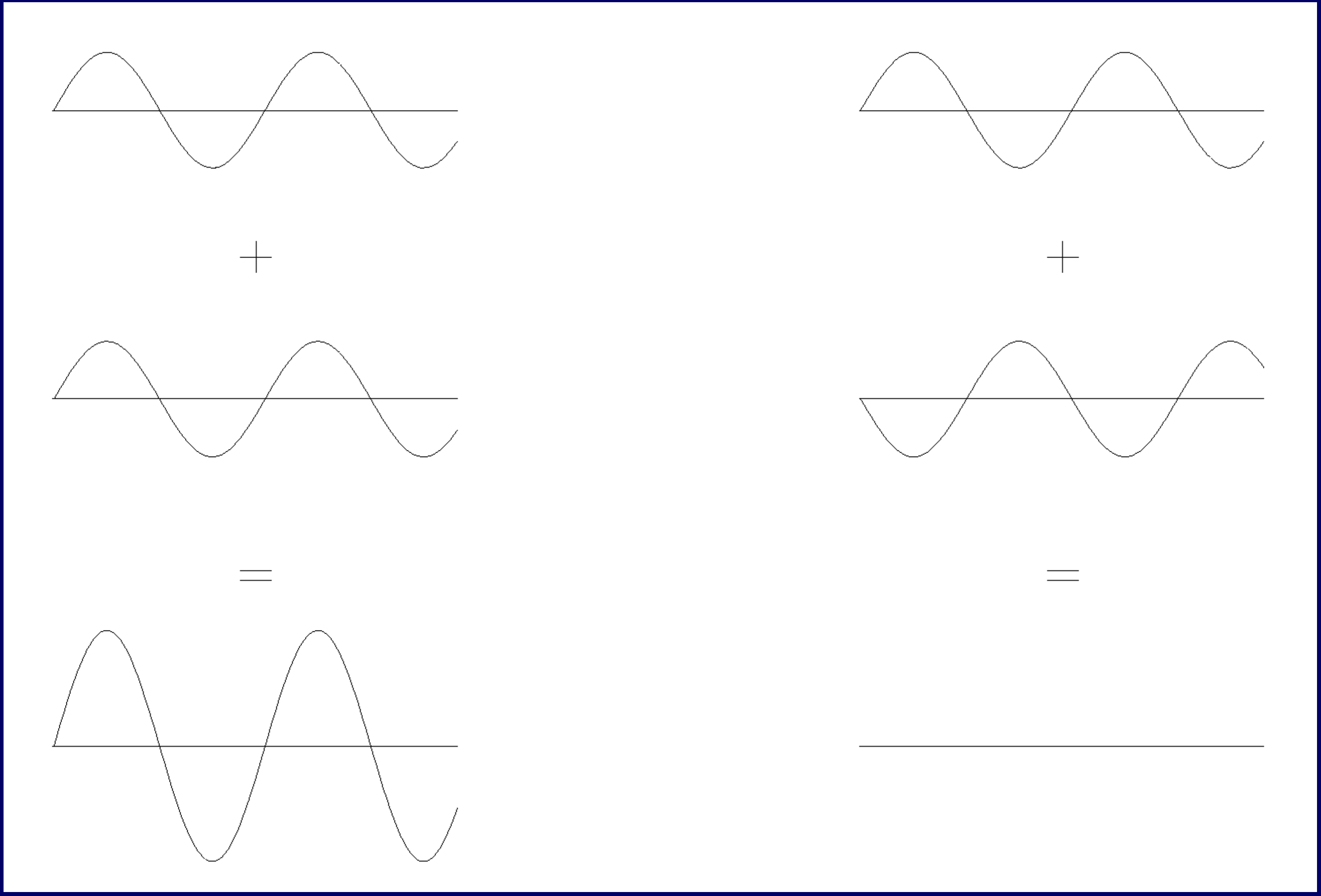
L >> d



Superposition

Constructive interference

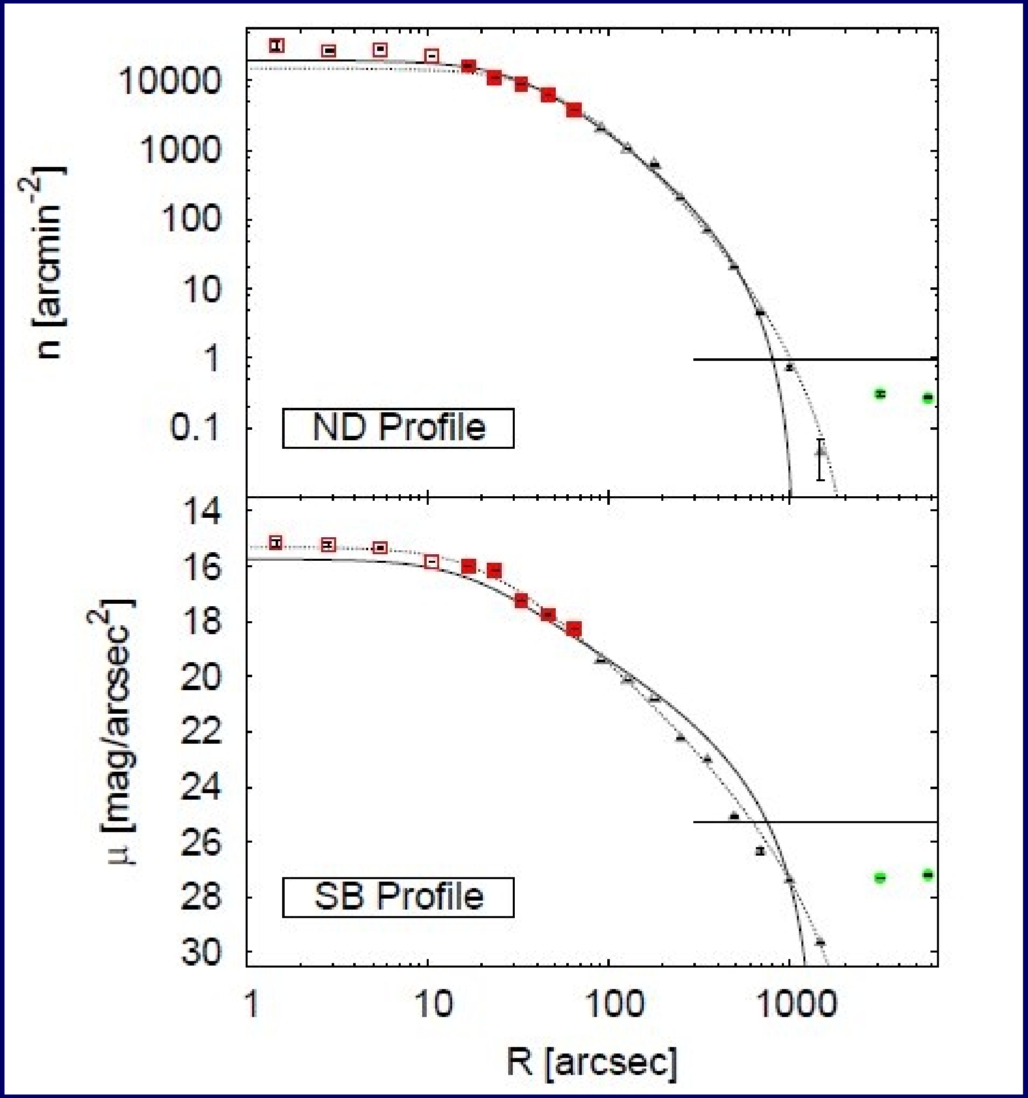
Destructive interference



Globular Cluster

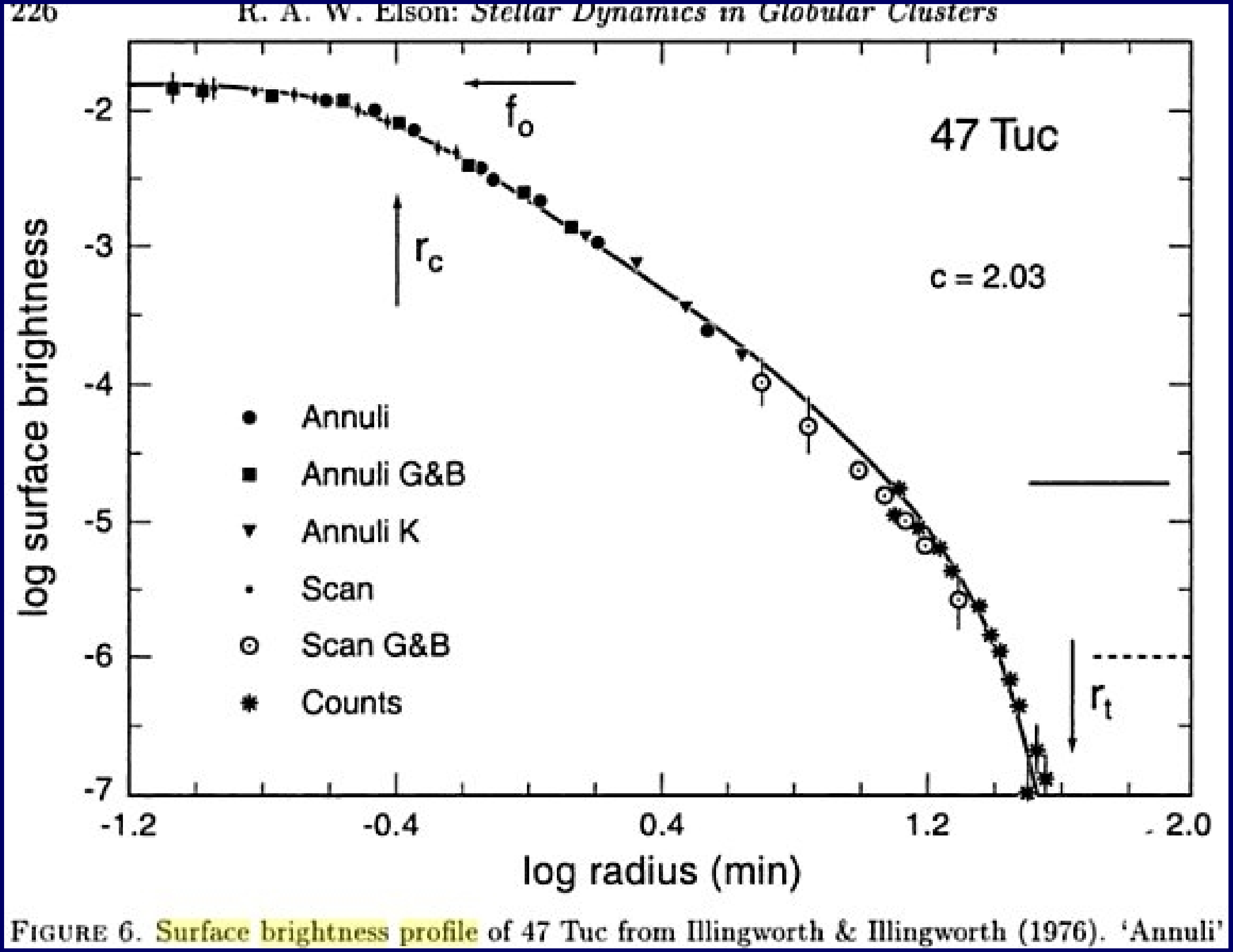
M92 in Hercules

Globular Cluster



M92 in Hercules

Globular Cluster



47 Tuc D=5 kpc

Poynting Vector, S

- A vector pointing in the direction of light propagation.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

- Magnitude is equal to the amount of energy per unit time that crosses a unit area oriented perpendicular to the direction of propagation of the wave
- Need to take a time average since wave vary harmonically with time (sin and cosine functions)
  - Average over one period: $$\langle S \rangle = \frac{1}{2\mu_0} E_0 B_0$$
- This is for a particular frequency
  - Need contributions from all frequencies to get the (bolometric) radiant flux.

NOTE: Light carries both energy and momentum, but does not have a rest mass.

Radiation Pressure

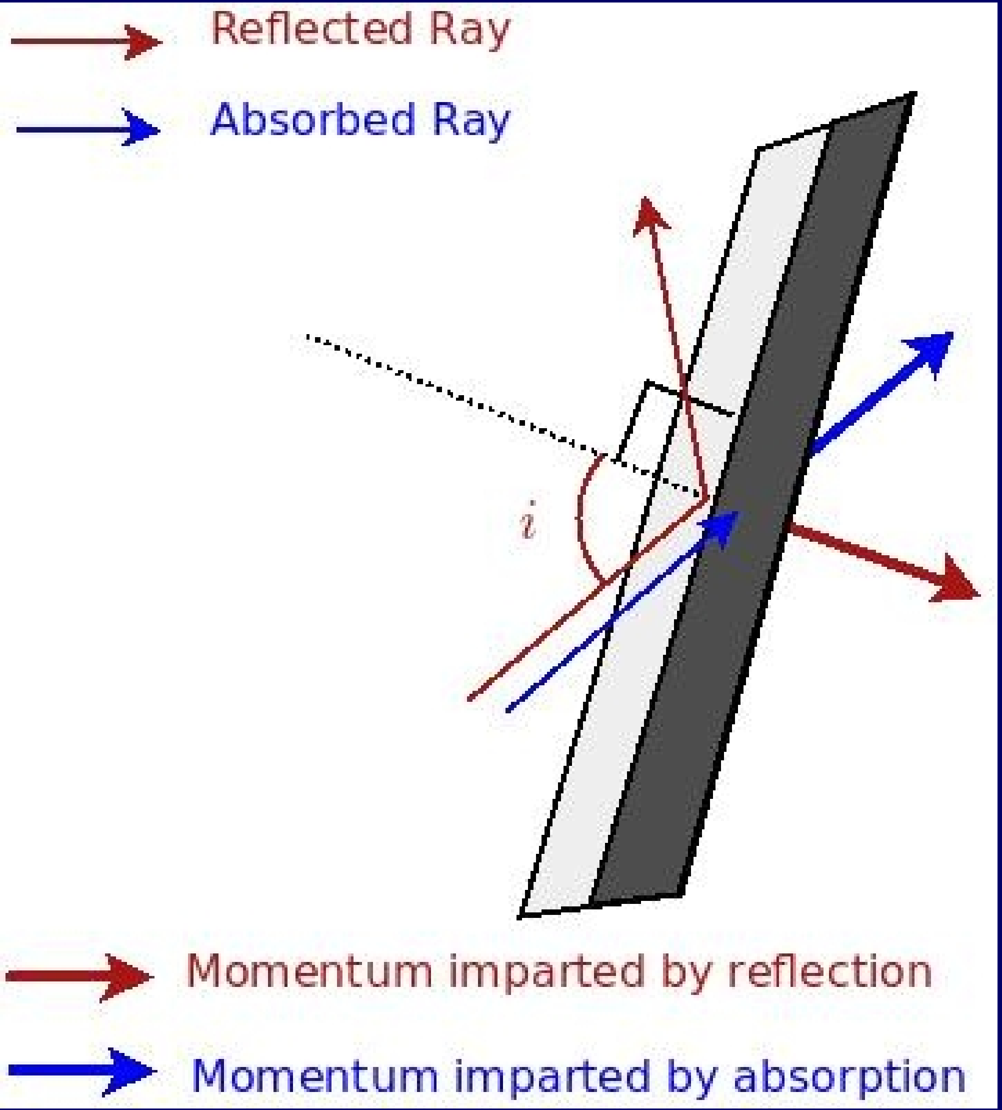
- Result of the momentum carried by the light.
  - Depends on reflection or absorption (both could happen)

Reflection:

$$P_{rad} = \frac{2 < S > \cos^2 \theta}{c}$$

Absorption:

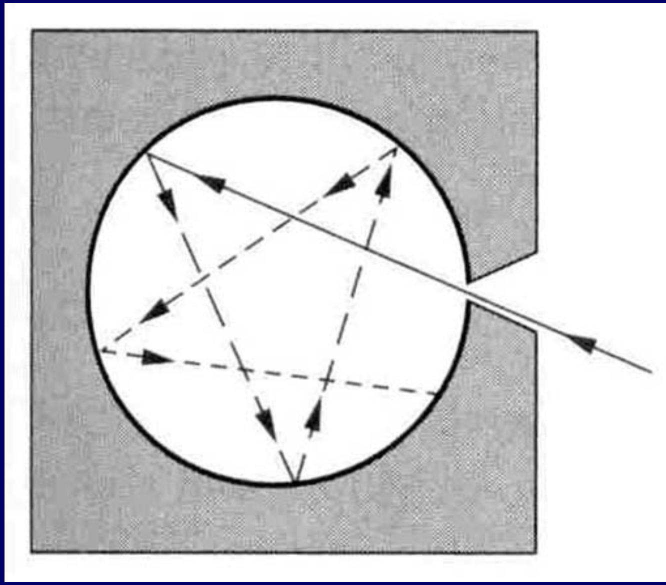
$$P_{rad} = \frac{1 < S > \cos^1 \theta}{c}$$



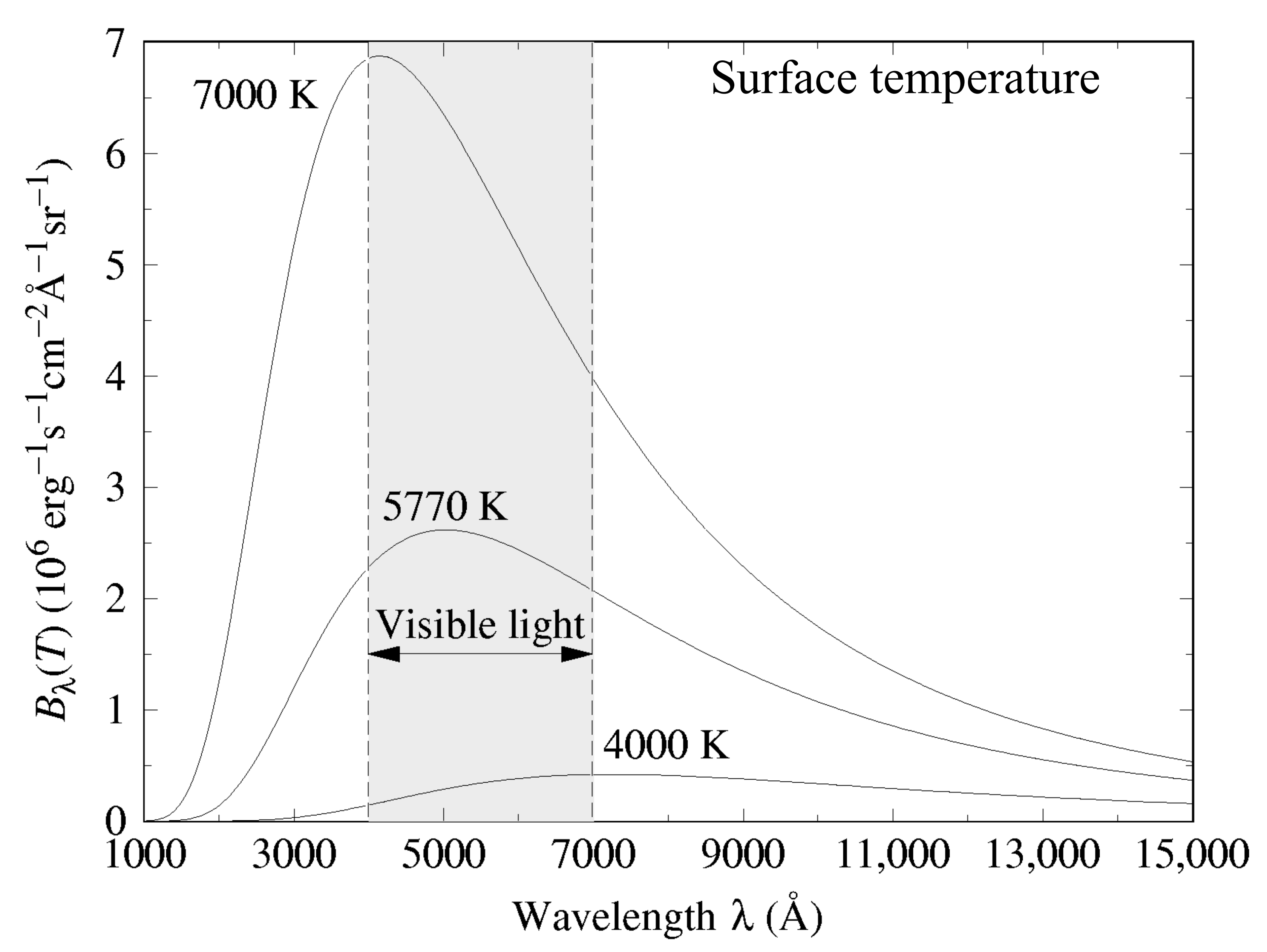
- Important role in stability of stars – equilibrium with gravity
- May also significantly effect interstellar "dust"

Blackbodies

- All objects above absolute zero emit radiation
  - Molecules and atoms are constantly in motion (thermal energy)
    - Accelerating charges → E & M radiation
  - Radiation (amount and type) will be temperature dependent
- Blackbody
  - Absorbs all incident radiation (no reflection)
  - Emits blackbody radiation, dependent on the objects temperature
- Stars and Planets are very close to being ideal blackbodies



Blackbody emission



Wien's Displacement Law

- Relates the surface temperature to the wavelength at the peak of the spectrum

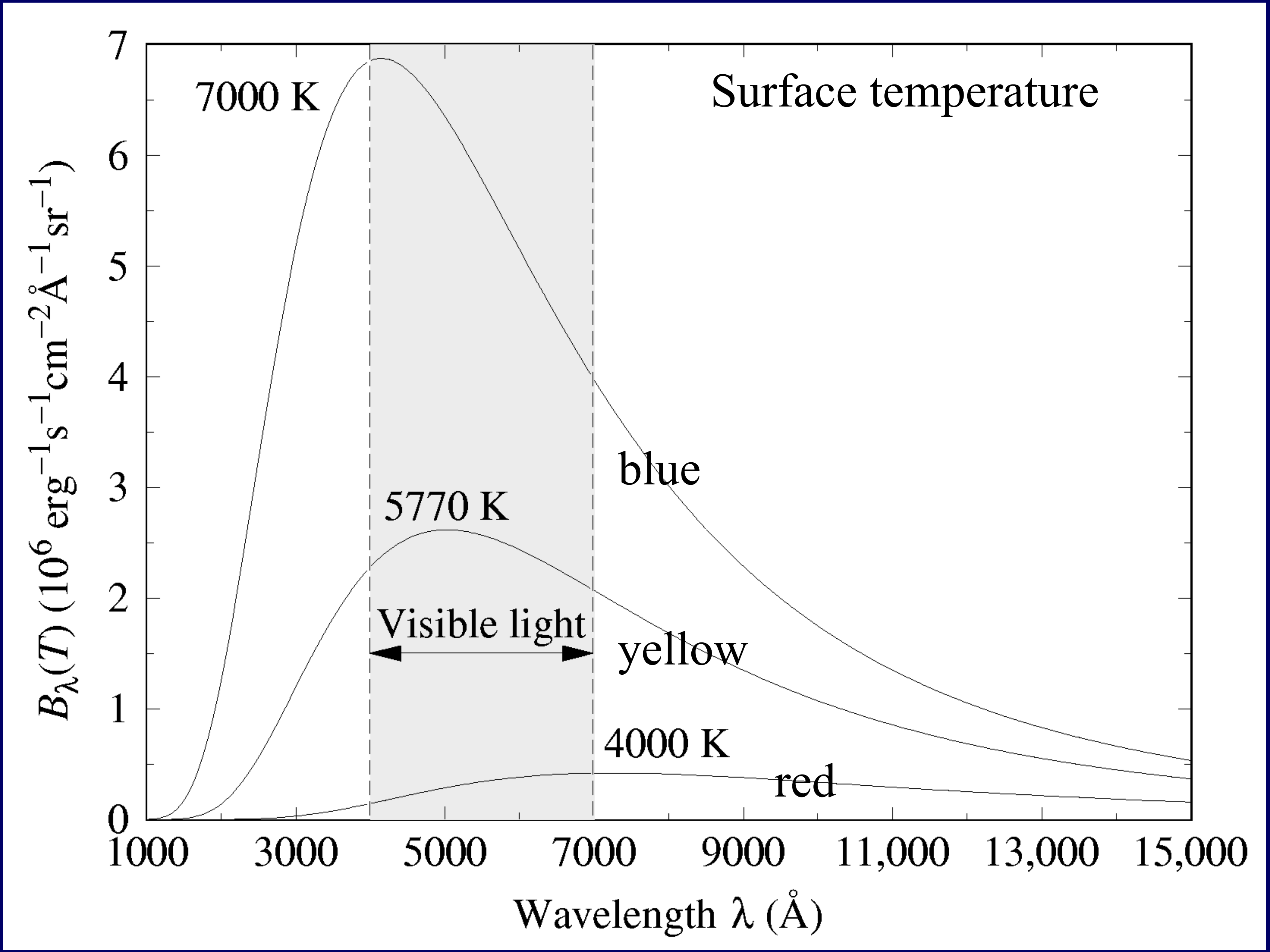
$$\lambda_{\max} T = 0.290 \text{ cm K}$$

- Helps explain the color of stars

- Alternatively written as

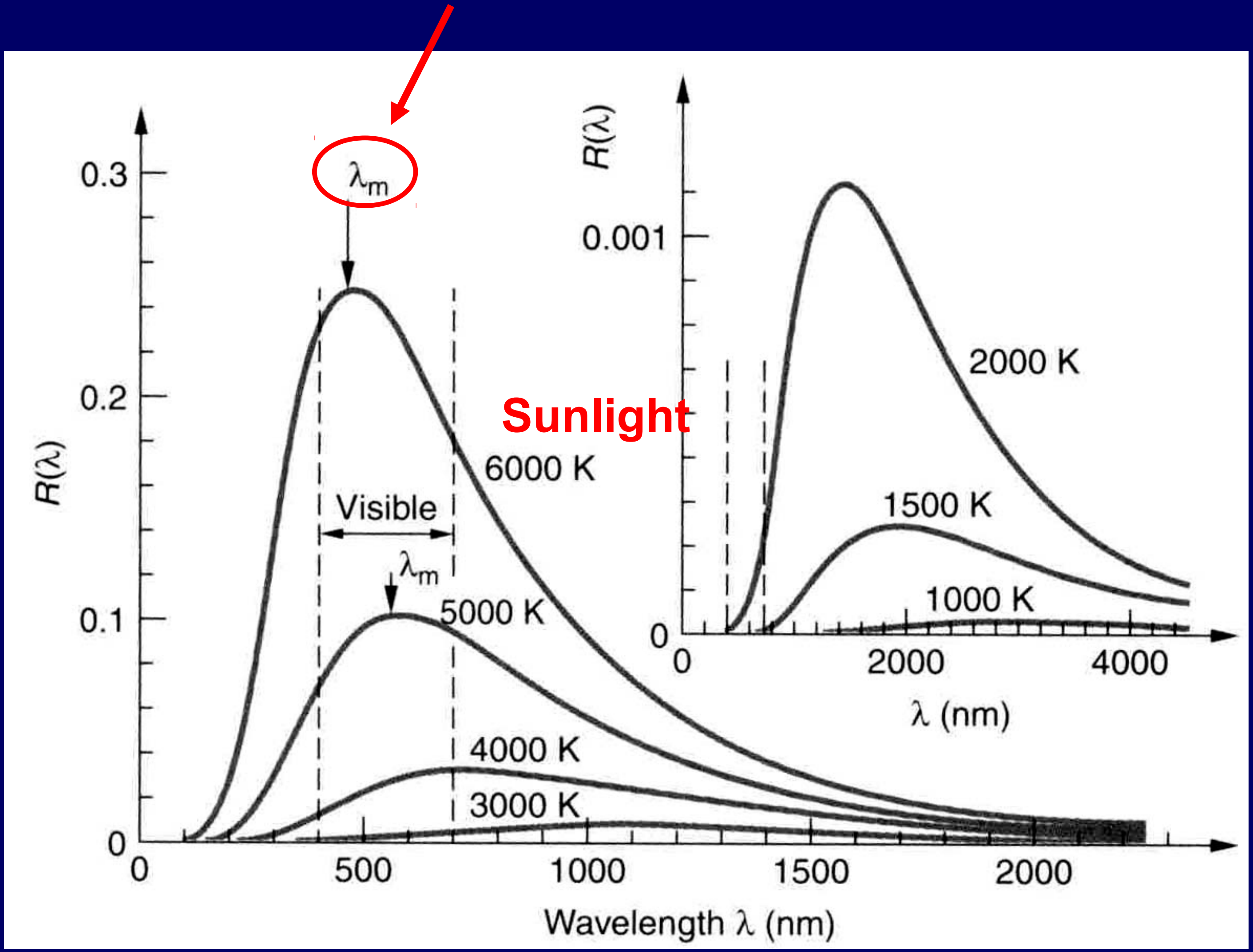
$$\lambda_{\max} T = (5000 \text{ Å})(5800 \text{ K})$$

Blackbody Emission



Blackbody: Simple Wien's Law

$$\lambda_{\max} = \frac{0.29}{T}\, cm \cdot K \propto \frac{1}{T}$$



Spectral Distribution depends only on Temperature

Stefan-Boltzmann Equation

- Total energy depends on temperature
- Empirically discovered by Stefan
- Derived from first principles (thermo and E&M laws) by Boltzmann
- Luminosity, L [ergs/sec]

$$L = \varepsilon \, \sigma \, A \, T^4$$

A = area of blackbody

T = Temperature in Kelvin

σ = Stefan-Boltzmann constant

$\mathcal{E}$ = emissivity, $0 \le \mathcal{E} \le 1$

$\mathcal{E} = 1$ is a perfect blackbody (usually assume this)

Stefan-Boltzmann Equation

- For a spherical blackbody <u>of radius *R*</u> the surface area is $4\pi R^2$, so the Luminosity is

$$L = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

- To get the <u>*radiant flux*</u> at the <u>surface</u> of the spherical blackbody, we divide by the area, $4\pi R^2$, so

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi R^2} = \sigma T^4$$

- At a known <u>distance, *d*,</u> from the BB, the radiant flux becomes

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi d^2} = \sigma T^4 \left(\frac{R}{d}\right)^2$$

- Since stars are not perfect blackbodies, the temperature is often called the *effective temperature* T_e (when $\varepsilon < 1$)

Spectral Blackbody: Distribution Definitions

u = Energy Density $$u(\lambda) = \frac{4\pi}{c} B_\lambda(T)$$ u is in ergs/cm^3

where B_λ = radiation emitted per unit time per unit area per unit wavelength in the unit solid angle $d\Omega$

Energy Density
Distribution Function

$$u(\lambda) = E_{ave}\ n(\ \ )$$

E_{ave} = average energy/mode (IMPORTANT QUANTITY)

$n(\lambda)$ = # oscillation modes = $8\pi\ \lambda^{-4}$ (independent of cavity shape)

#modes → count the standing waves

boundary conditions, nodes at the walls (E field = 0)

Spectral Blackbody: Rayleigh-Jeans Equation

$$u(\lambda) = kT \frac{8\pi}{\lambda^4}$$

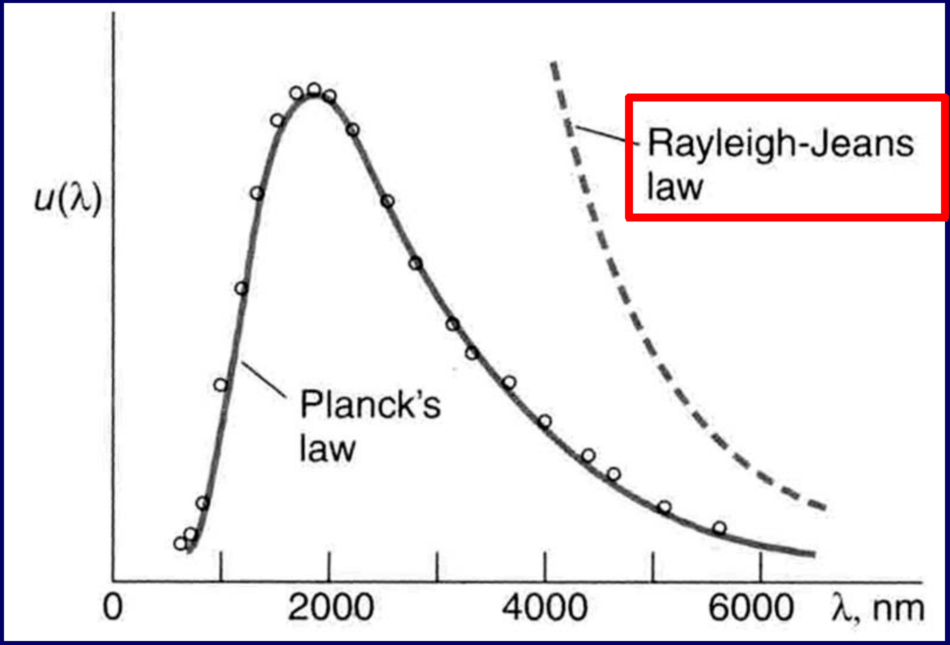
$E_{ave} = kT$ (Boltzmann distribution) and $n(\lambda) = 8\pi\lambda^{-4}$

$k = 1.38 \times 10^{-23}$ J/K (Boltzmann's constant)

$$\int_0^\infty u(\lambda)\, d \quad \rightarrow \infty$$

UV Catastrophe!

(explodes for small λ)



Spectral Blackbody: Planck's Law

- Planck's Law initially found empirically (trial and error!)
- Quantize the E&M radiation (photons)
  - Minimum energy

$$E_v = hv = hc/\lambda$$

where h = Planck's Constant = 6.266 x 10^{-27} erg·s

- This is used in replacing the classical *kT* expression for the average energy in a mode

$$E_v = nhv, \quad n = 0, 1, 2, 3$$

  - Avoids the catastrophe – the entire hot object does not have enough energy to emit one quanta of EM waves

Spectral Blackbody: Derivation of Planck's Law

- **OLD** (Classical from Boltzmann/ Raleigh-Jeans)

$$f(E) = Ae^{\frac{-E}{kT}}$$

$$E_{ave} = \int EAe^{\frac{-E}{kT}}dE = kT$$

- **NEW** (Quantum from Planck)

$$f_n(E_n) = Ae^{\frac{-En}{kT}}$$

$$E_{ave} = \sum E_n Ae^{\frac{-E_n}{kT}} = \frac{(hc/\lambda)}{e^{\frac{hc/\lambda}{kT}} - 1}$$

where $E_n = nhv$

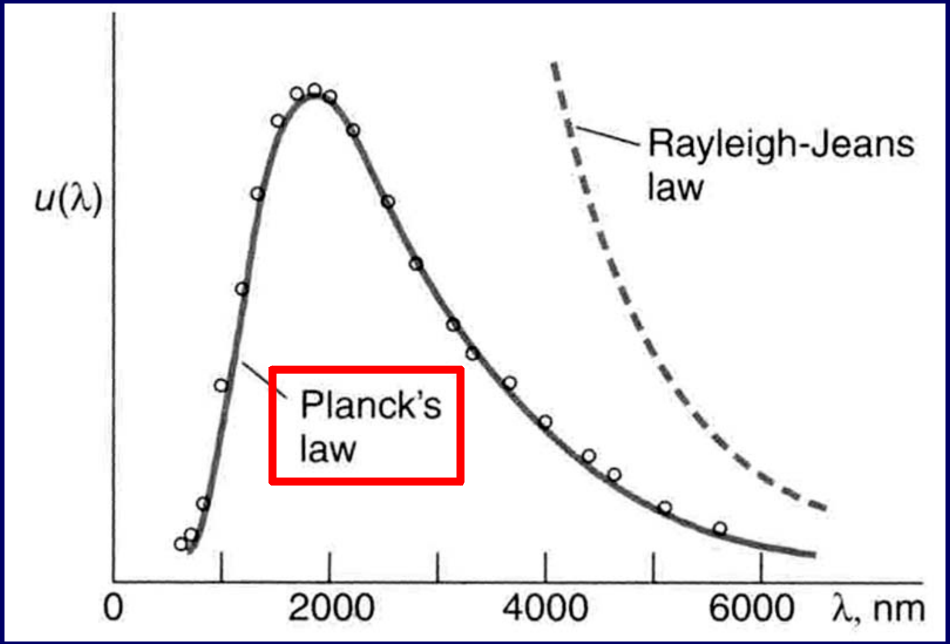
- Assumption of **Quantization is CRITICAL!**

Spectral Blackbody: Planck's Law

- Planck's Law initially found empirically (trial and error!)
- Quantize the E&M radiation (photons)

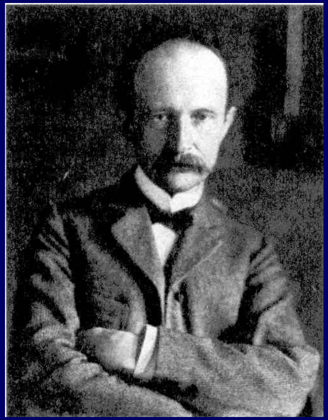
$$E_\nu = h\nu = hc/\lambda$$

$$u(\lambda) = \frac{(hc/\lambda)(8\pi\lambda^{-4})}{e^{\frac{hc/\lambda}{kT}} - 1}$$



- $E_{ave} = (h\nu)[\exp(h\nu/kT) - 1]^{-1}$ and $n(\lambda) = 8\pi\lambda^{-4}$
- Energy of a photon: $\mathbf{E = hc/\lambda}$ and $c = \nu\lambda$

Spectral Blackbody: Limits of Planck's Law



$$u(\lambda) = \frac{\left(hc/\lambda\right)\left(8\pi\lambda^{-4}\right)}{e^{\frac{hc/\lambda}{kT}} - 1}$$

- Limit of **Large λ** (or small energy E)

$$e^{\frac{hc/\lambda}{kT}} \approx 1 + \frac{hc/\lambda}{kT} + \ldots$$

Taylor's Series for small exponent

$$u(\lambda \rightarrow \infty) \rightarrow \left(8\pi\lambda^{-4}\right) kT$$

Rayleigh-Jeans Equation

- Limit of **Small λ** (or large energy E)

$$u(\lambda \rightarrow 0) \rightarrow \lambda^{-5} e^{\frac{-hc/\lambda}{kT}} \rightarrow \boxed{0}$$

Planck's Law

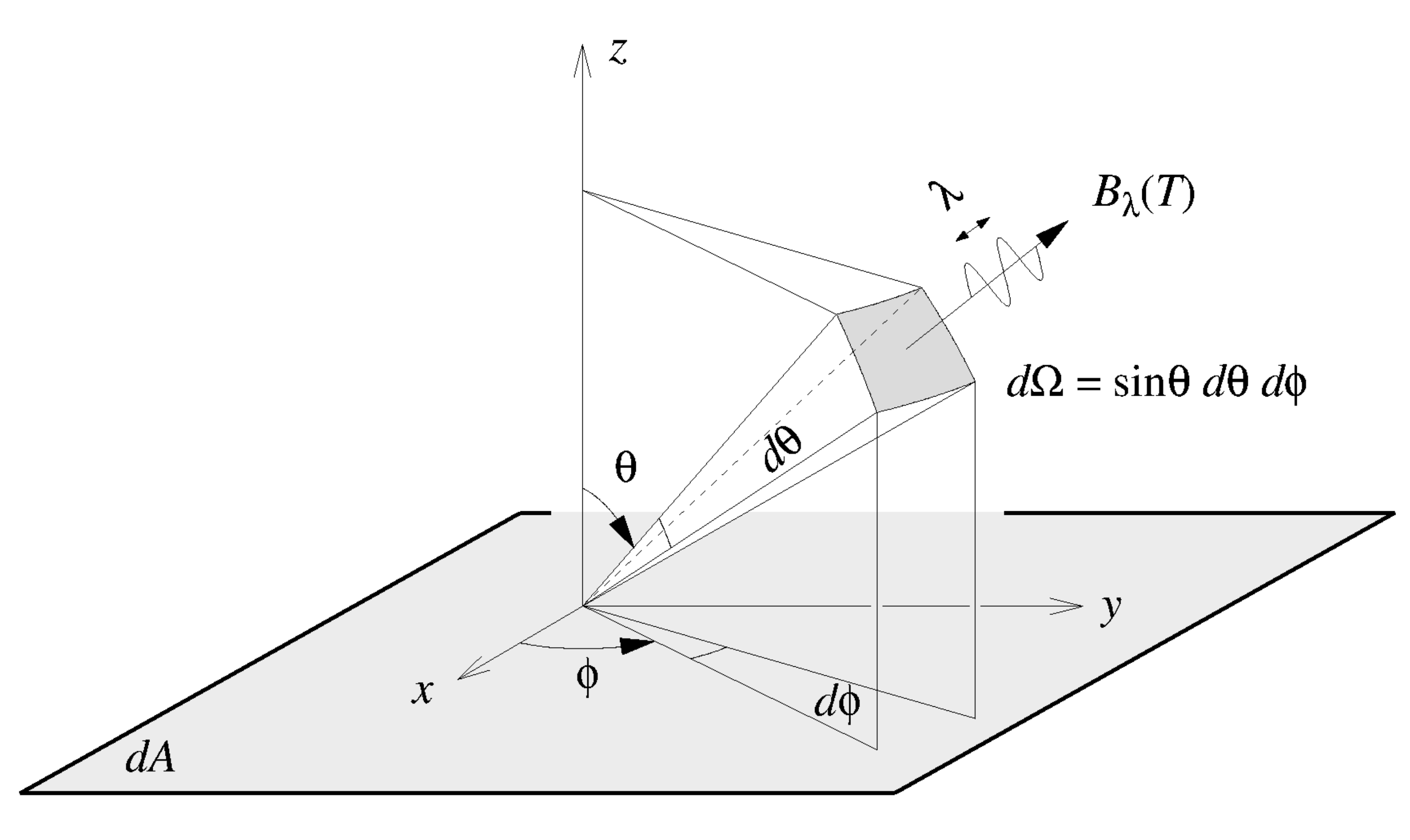
$$B_\lambda(\lambda, T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$B_\nu(\nu, T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Energy from a surface element.

B_λ Units:

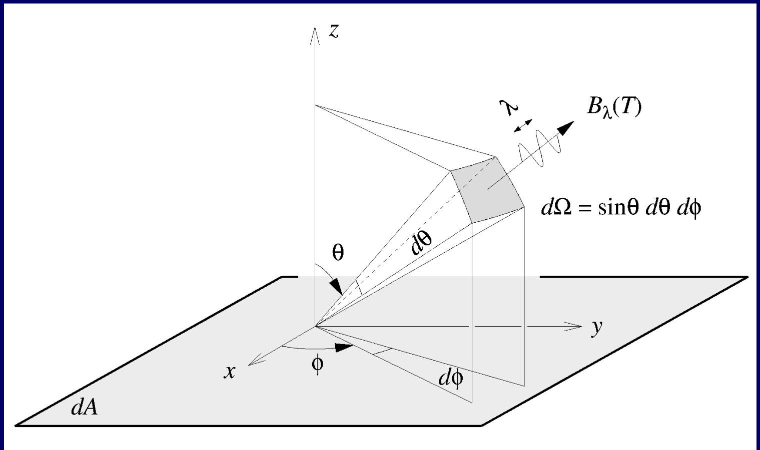
$$\frac{erg}{cm^2 \cdot s \cdot d\Omega \cdot d\lambda}$$



Planck's Law

- Observations: Radiant flux and apparent magnitude
- Star Properties: Radius, temperature
- Need to integrate over:
  - Area (sphere)
  - solid angle (from the *flat* infinitesimal surface element)
    - Isotropic – no preferred direction
- Monochromatic Luminosity

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$



$$L_\lambda d\lambda = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \left(B_\lambda \, d\lambda\right)\left(dA \, \cos\theta\right)\left(\sin\theta \, d\theta \, d\varphi\right)$$

$$4\pi^2 R^2 B_\lambda \, d\lambda$$

From λ to λ + *d*λ

The Color Index

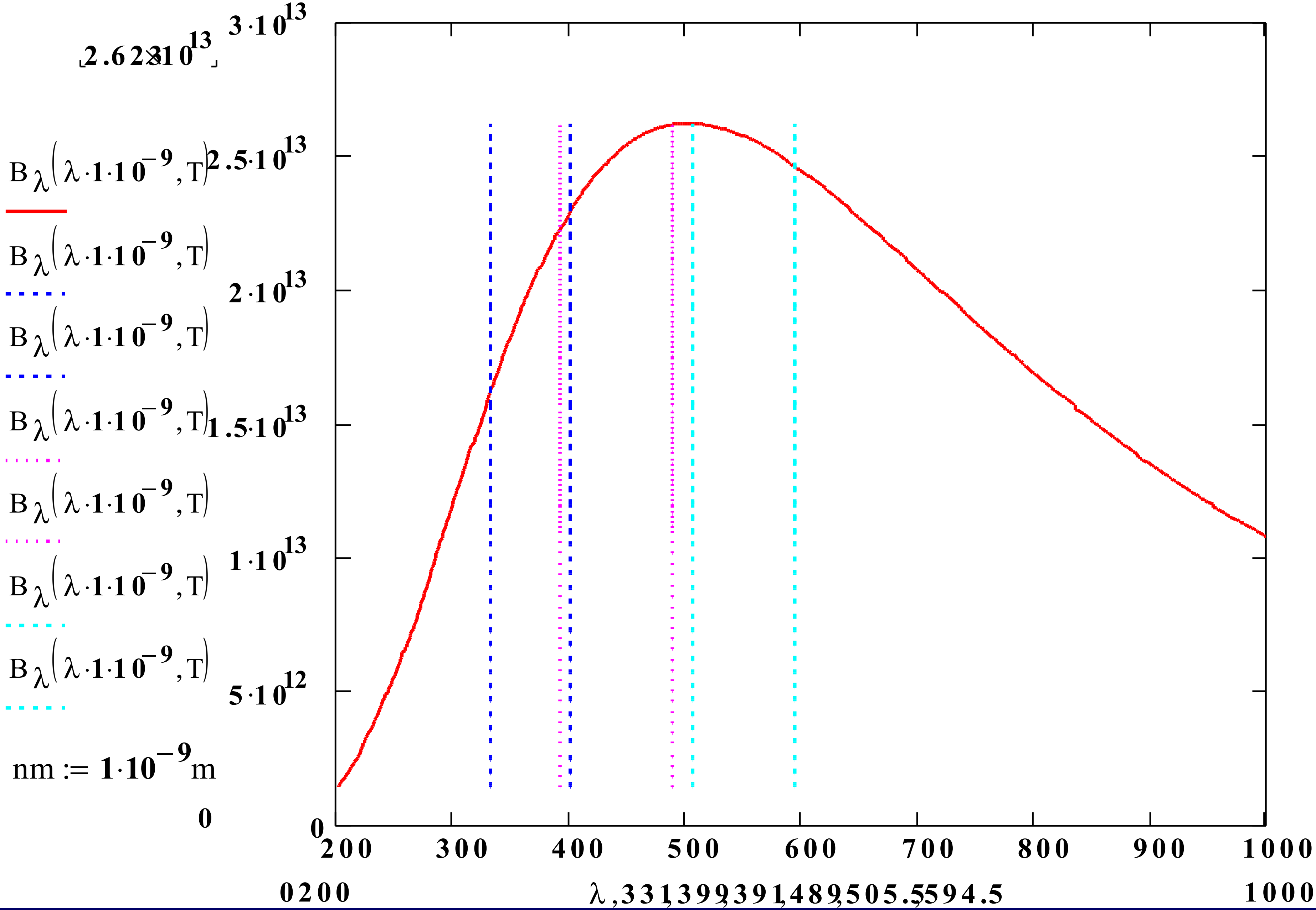
- M_{bol} or m_{bol} is at all wavelengths and is called the bolometric magnitude
- Monochromatic flux integrated over a wavelength range
- Standard filters for the UBV system (there are other systems)
  - U is ultraviolet $\lambda_{center} = 3650$ Å

    $\Delta\lambda = 680$ Å
  - B is blue $\lambda_{center} = 4400$ Å

    $\Delta\lambda = 980$ Å
  - V is visible $\lambda_{center} = 5500$ Å

    $\Delta\lambda = 890$ Å

The Color Index



The Color Index

- Knowing the distance, the absolute color magnitudes can be determined, M_U, M_B, M_V.

$$m - M = 5\log_{10}\left(\frac{d}{10\ pc}\right)$$

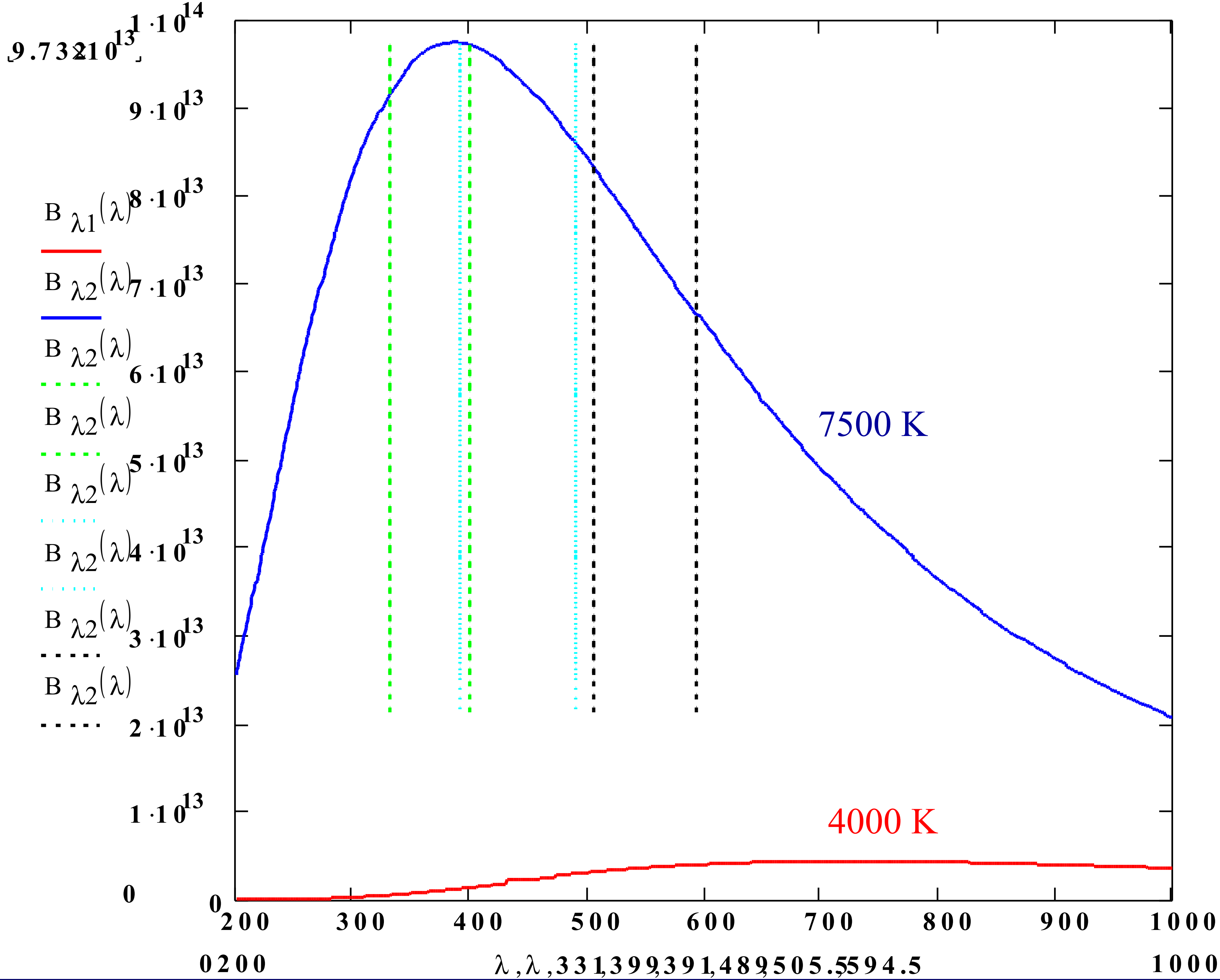
- Apparent magnitudes are: U, B, and V (instead of *m*)
- Color Indices
  - Independent of distance!

$$U - B = M_U - M_B$$

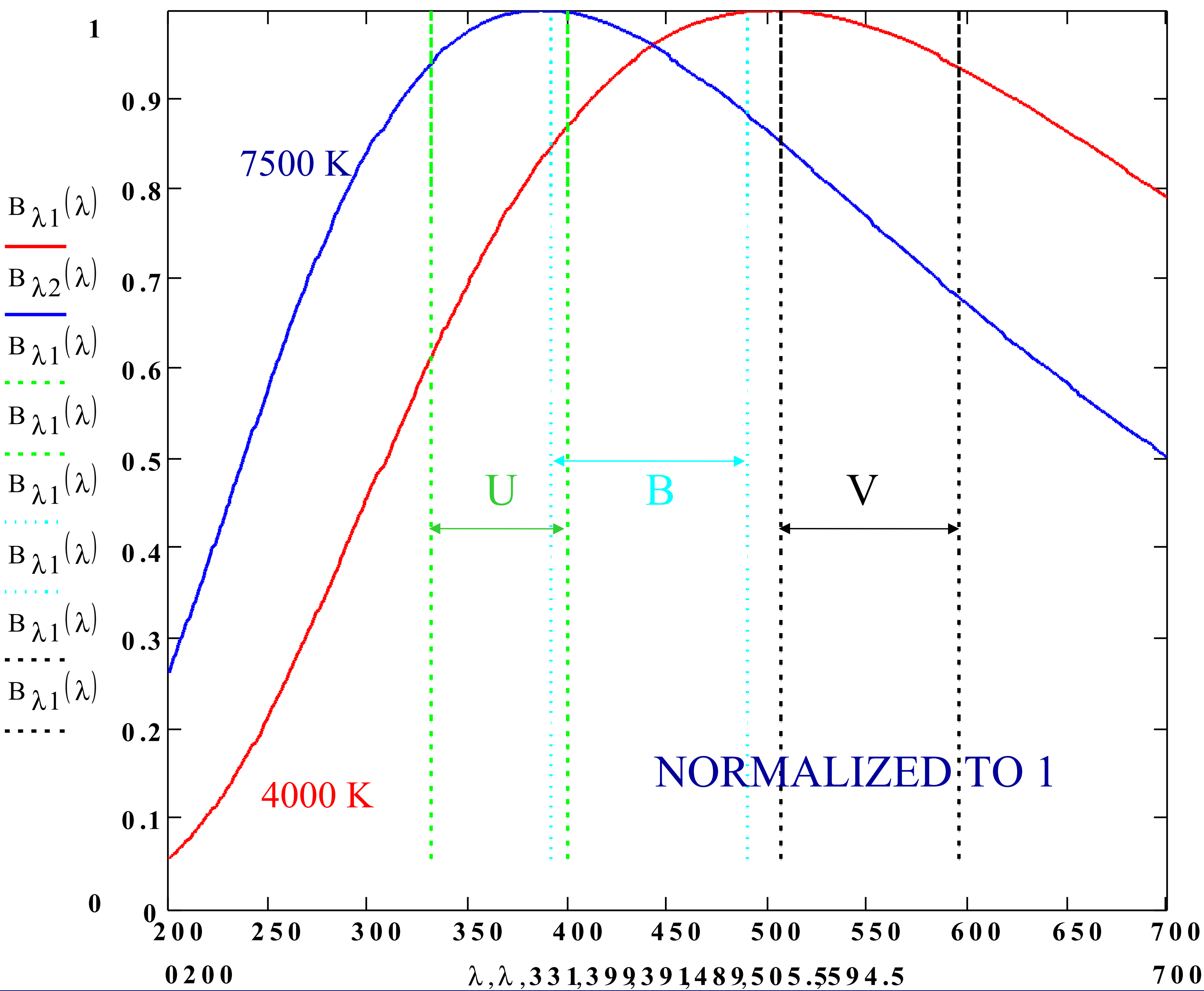
$$B - V = M_B - M_V$$

- Smaller (B-V) is bluer
  - Stellar magnitudes decrease with increasing brightness
- Bolometric correction, BC:

$$BC = m_{bol} - V = M_{bol} - M_V$$



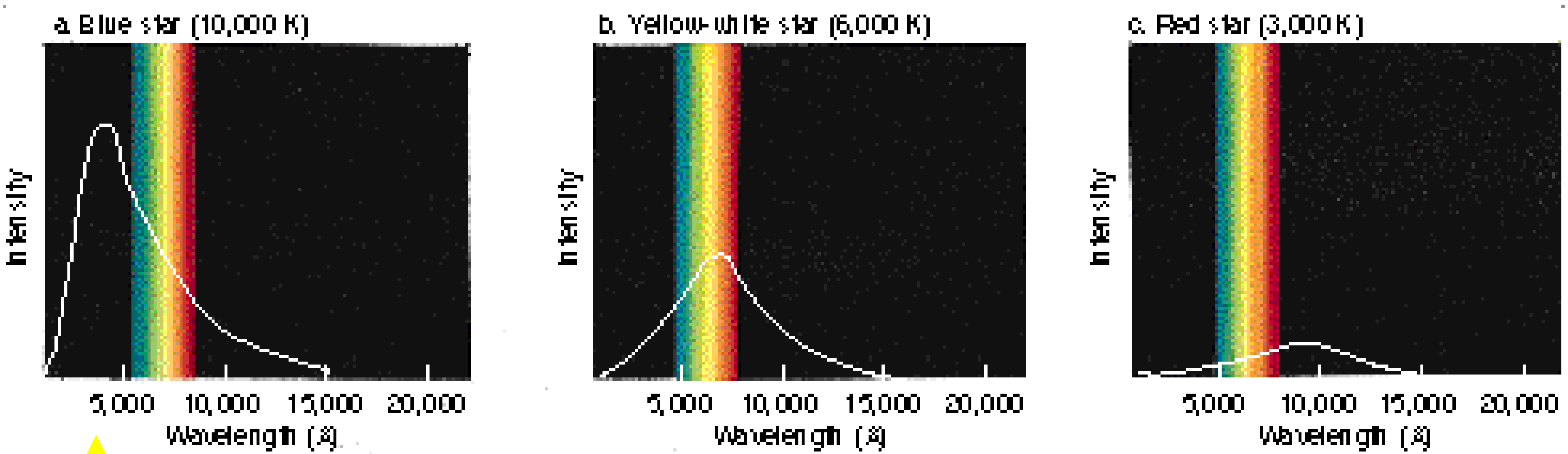
7500 K
4000 K
B λ1(λ)
B λ2(λ)
1·10^14
9.7321·10^13
9·10^13
8·10^13
7·10^13
6·10^13
5·10^13
4·10^13
3·10^13
2·10^13
1·10^13
0
200
300
400
500
600
700
800
900
1000
λ, λ, 331, 399, 391, 489, 505.5, 594.5



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
7500 K
4000 K
U
B
V
NORMALIZED TO 1
200 250 300 350 400 450 500 550 600 650 700
0 200
λ, λ, 331, 399, 391, 489, 505.5, 594.5
700

Temperature of Stars

- Determined by type of em radiation



Peak in UV

Peak in visible

Peak in IR

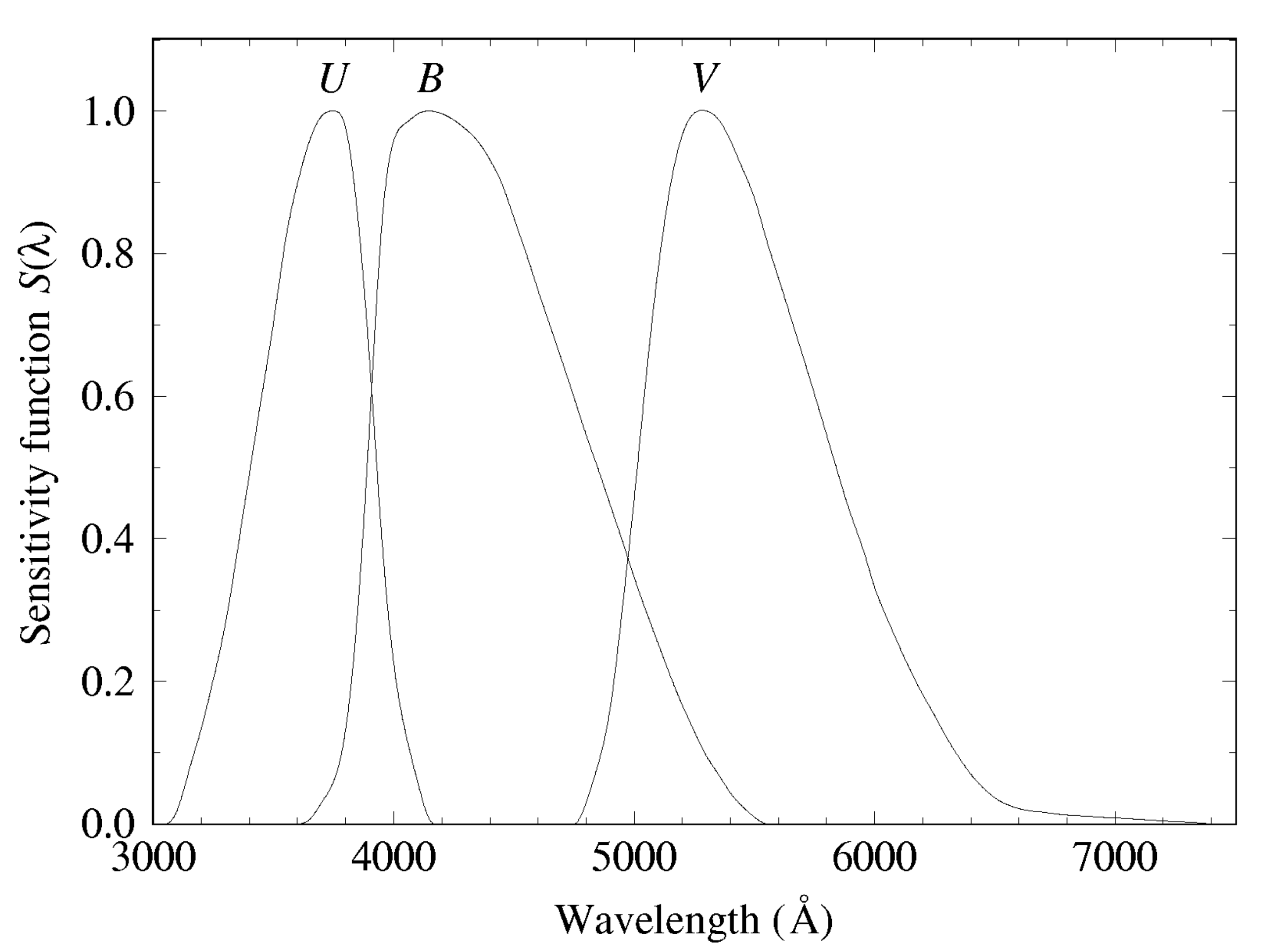
The Color Index

- Sensitivity Functions, $S(\lambda)$

$$m_1 - m_2 = -2.5\log_{10}\left(\frac{F_1}{F_2}\right)$$

$$U = -2.5\log_{10}\left(\int_0^\infty F_\lambda S_U \, d\lambda\right) + C_U$$

$U = 0$ for Vega determines C_U



Example 3.6 – Sirus

- Brightest star in the sky

  U = -1.50, B = -1.46, V = -1.46

  $U - V$ = -0.04

  $B - V$ = 0.00

  Brightest at UV wavelengths, T_e = 9910 K:

$$\lambda_{\max} = \frac{0.29\ cm \cdot K}{9910\ K} = 2.926 \times 10^{-5}\ cm = 2.926 \times 10^{-7}\ m = 2926\ \mathring{A}$$

  Bolometric correction is: BC = -0.09, so its apparent bolometric magnitude is $m_{bol} = V + BC = -1.46 + (-0.09) = -1.55$

  (this is brighter than Vega)