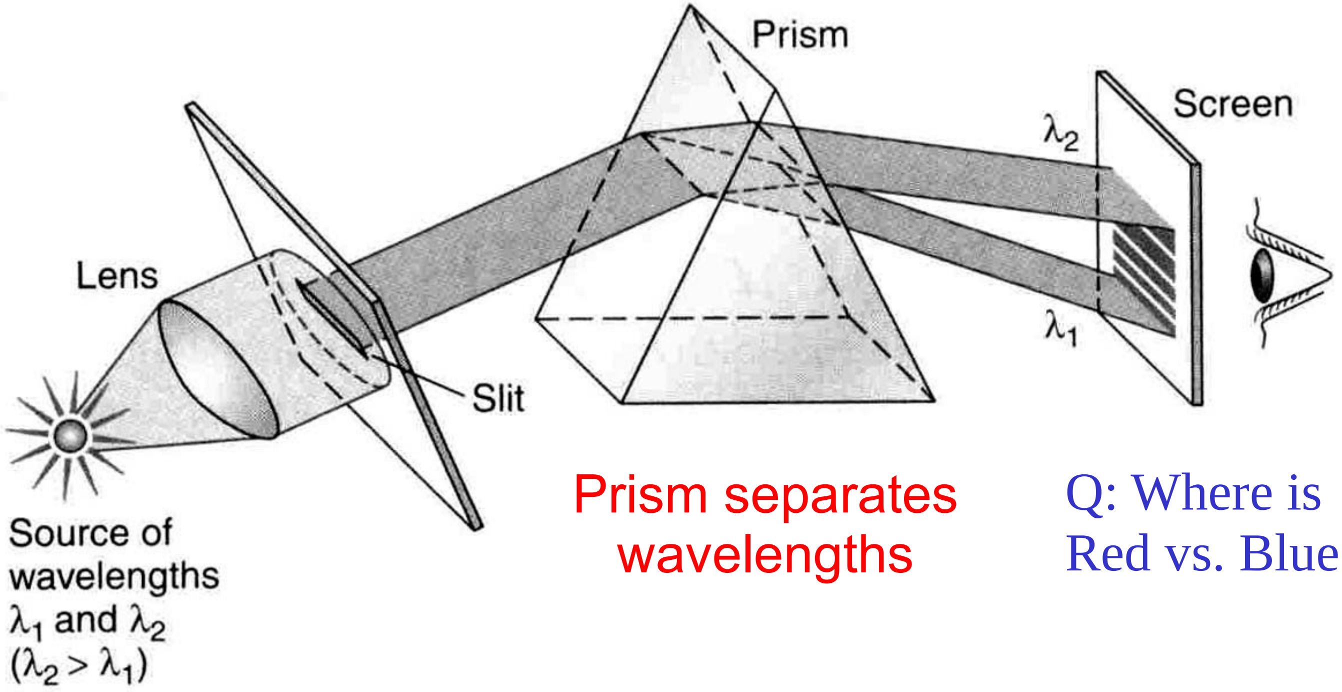
The Interaction of Light and Matter

Outline

- (1) Motivation: Why spectral lines?
  - the Birth of Spectroscopy
  - Kirchoff's Laws
- (2) Photons – the particle nature of light
  - Blackbody radiation (Planck introduces quantum of light)
  - Photoelectric Effect
  - Compton Scattering
- (3) The Bohr Model of the Atom
  - a theory to describe spectral lines,
- (4) Quantum Mechanics and the Wave-Particle Duality (SKIP on ExamI)
  - De Broglie wavelength
  - Schrodinger's probability waves.

Spectroscopy - history

- Trogg (50 million BC) – rainbow
- Newton (1642-1727) – decomposes light into spectrum and back again
- W. Herschel (1800) – discovers infrared
- J. W. Ritter (1801) – discovers ultraviolet
- W. Wollaston (1802) – discovers absorption lines in solar spectrum



Prism separates wavelengths

Q: Where is Red vs. Blue?

Spectroscopy - history

- J. Herschel, Wheatstone, Alter, Talbot and Angstrom studied spectra of terrestrial things (flames, arcs and sparks) ~1810
- Joseph Fraunhofer
  - Cataloged ~475 dark lines of the solar spectrum by 1814
  - Identifies sodium in the Sun from flame spectra in the lab!
  - Looks at other stars (connects telescope to spectroscope)
- Foucault (1848) – sees absorption lines in sodium flame with bright arc behind it.

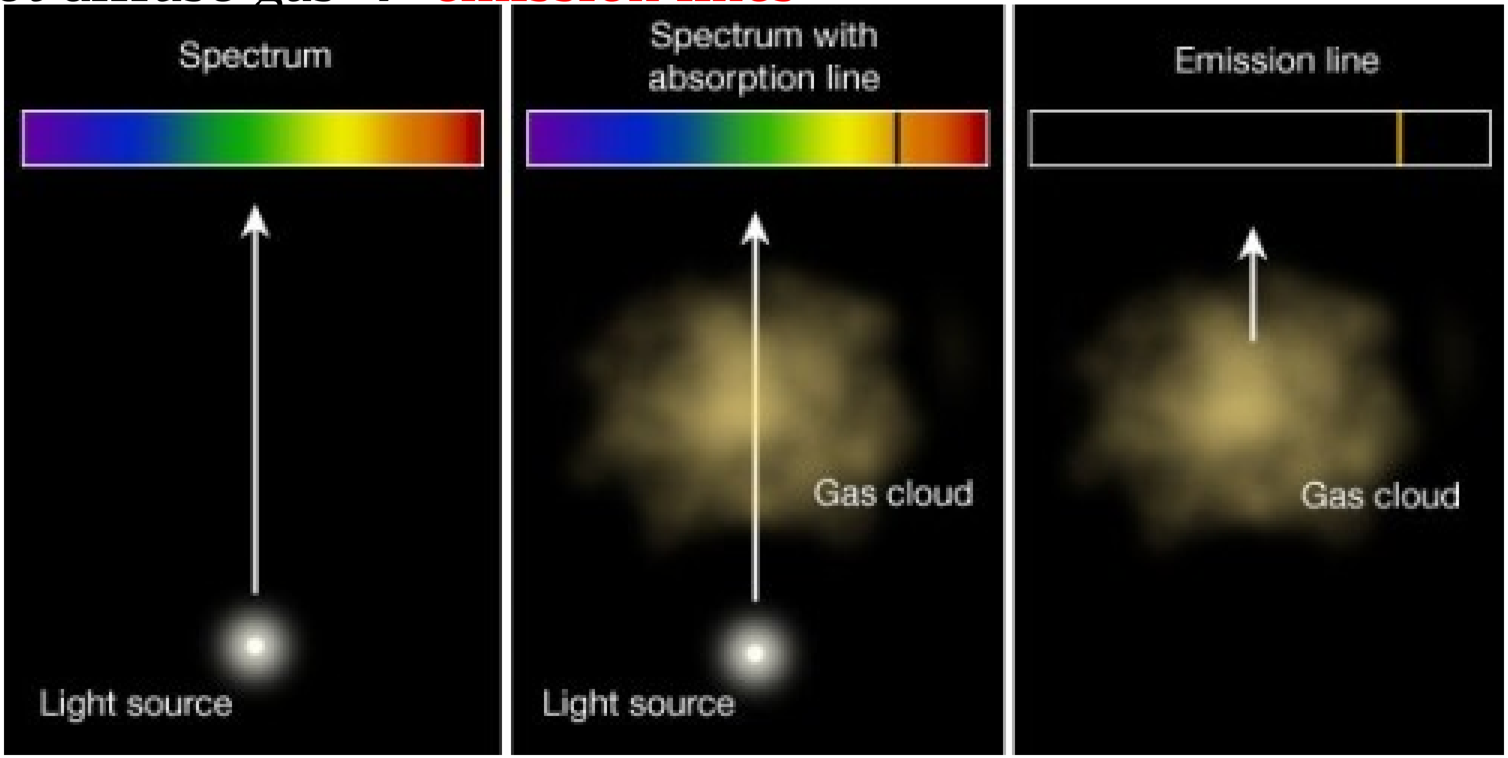
There was no accepted explanation for the absorption lines. *New physics* needed*!*

Kirchhoff's laws (1859):

Kirchhoff worked with Bunsen on flame spectra

Developed a prism spectroscope

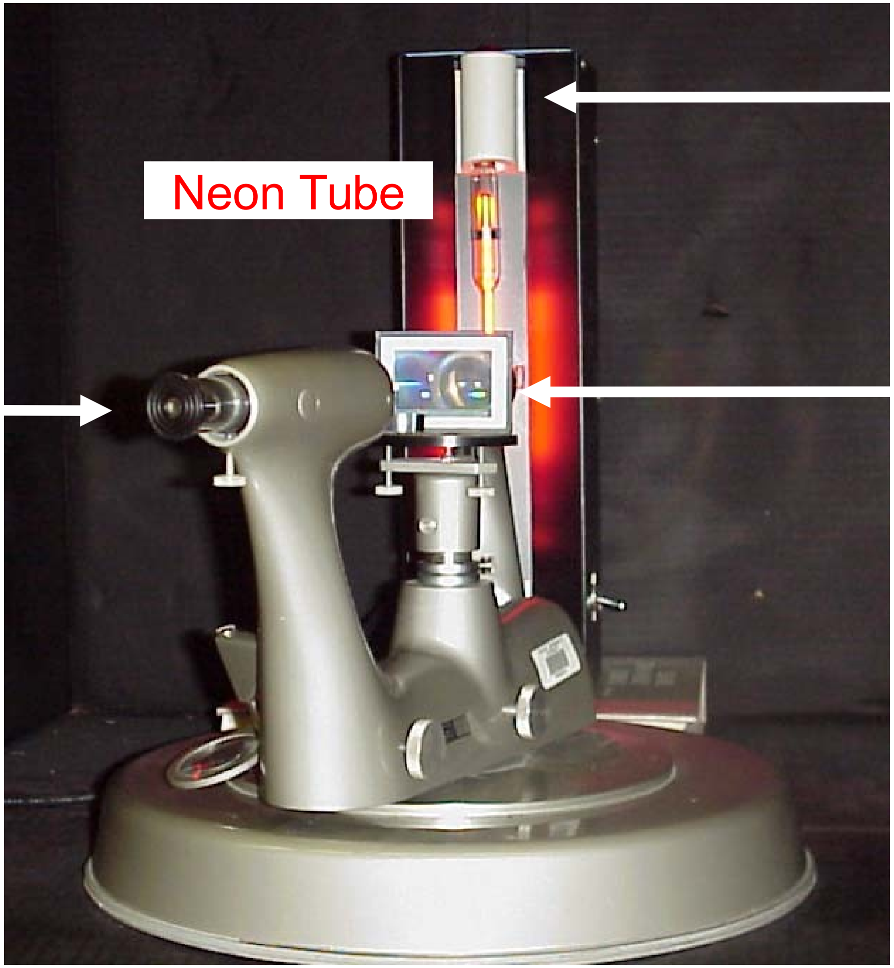
- Hot solid or dense gas, → continuous spectrum (eg Blackbody)
- Cool diffuse gas in front of a blackbody → absorption lines
- Hot diffuse gas → emission lines



Spectroscope for typical atomic physics lab

High Voltage Supply

Eyepiece



Diffraction Grating

$$d \sin \theta = n\lambda$$

Doppler shift (see also Ch. 4)

- Spectral lines allow for the measurement of radial velocities
- At low velocities, $v_r << c$
  - Classical Doppler effect
    - *Radial velocity*, v_r
    - *Heliocentric correction* for Earth's motion, up to 29.8 km/s, depending on direction.

$$\frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rst}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v_r}{c}$$

$$\Delta\lambda = \frac{v_r}{c} \lambda_{rest}$$

- Example: H_α is 6562.80 Å
  - Vega is measured to be 6562.50 Å
  - Coupled with the *proper motion*
    - Can determine total velocity

$$v_r = c\frac{\Delta\lambda}{\lambda_{rest}} = -14\frac{km}{sec}$$

$$v_\theta = r\mu = 13\frac{km}{s}$$

$$v = \sqrt{v_r^2 + v_\vartheta^2} = 19\frac{km}{s}$$

Doppler shift

- Since most galaxies are moving away, astronomers call the Doppler shift a *redshift*, *z*.

$$z = \frac{\Delta\lambda}{\lambda_{rest}}$$

- At high velocities, v_r <~ c
  - Relativistic redshift parameter (Ch. 4):

$$z = \sqrt{\frac{1 + v_r/c}{1 - v_r/c}} - 1$$

  - Example: Prob. 4.8.
    (should get: $v_r = 0.9337c$)

Particle/Wave Duality - Part 1

PART 1

- **Electrons** as discrete **Particles**
  - Measurement of **e/m** (CRT) and **e** (oil-drop expt.)
- **Photons** as discrete **Particles**
  - **Blackbody Radiation**: Temp. Relations & Spectral Distribution
  - **Photoelectric** Effect: Photon "kicks out" Electron
  - **Compton** Effect: Photon "scatters" off Electron

PART 2

- **Wave Behavior:** Diffraction and Interference
- **Photons** as **Waves**: $\lambda = hc / E$
  - X-ray Diffraction (Bragg's Law)
- **Electrons** as **Waves**: $\lambda = h / p$
  - Low-Energy Electron Diffraction (LEED)

Photons: Quantized Energy Particle

- Light comes in discrete energy "packets" called photons

Energy of Single Photon

$$E = h\nu = \frac{hc}{\lambda}$$

From Relativity:

$$E^2 = (pc)^2 + (mc^2)^2$$

Rest mass (m)

For a Photon (m = 0):

$$E^2 = (pc)^2 + 0 \Rightarrow \boxed{E = pc}$$

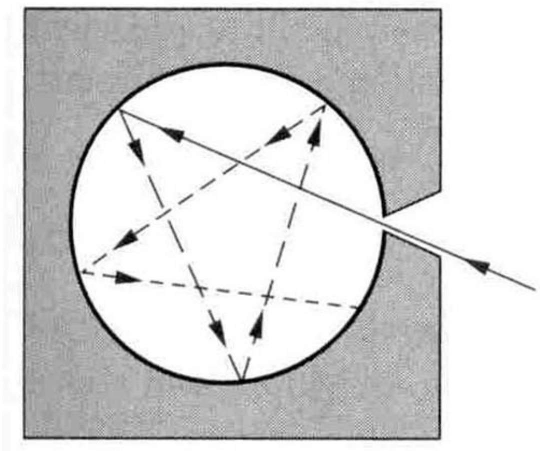
Momentum of Single Photon

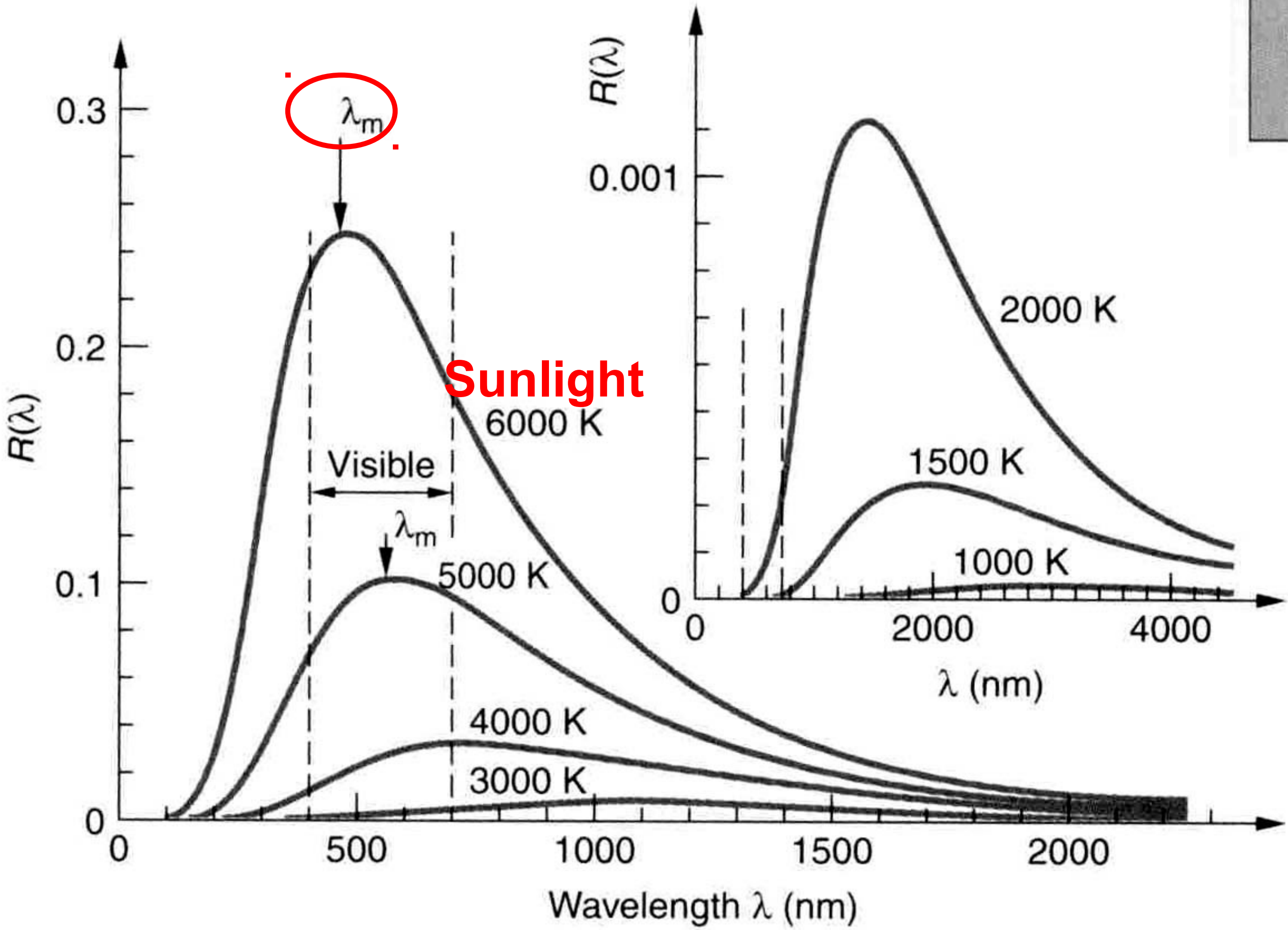
$$p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

Blackbody Radiation: First clues to quantization

Recall Wien's Law: $$\lambda_{max} = \frac{0.029}{T} cm \cdot K$$

and the Stefan-Boltzmann Law: $$F = \sigma T^4$$





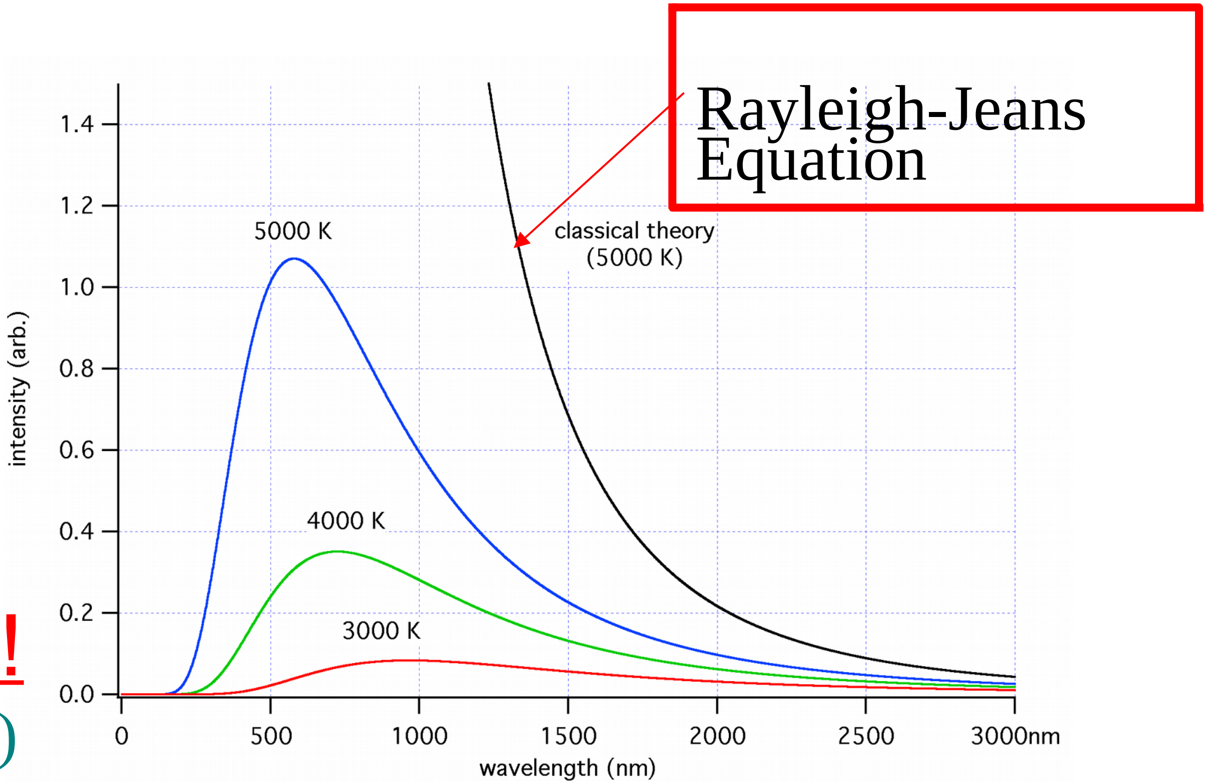
Spectral Distribution depends only on Temperature

Blackbody Radiation: Rayleigh-Jeans Equation

Classical physics led to a prediction for the spectrum of cavity (blackbody) radiation whereby $B_{\lambda}(T) = \frac{2ckT}{\lambda^4}$

This was derived by assuming each mode of oscillation in the cavity would have an energy $E_{avg} = kT$ (k = 1.38 x 10^{-23} J/K is Boltzmann's constant)

The number of modes per wavelength interval increased as lambda decreased leading to excess energy at small λ s.



UV Catastrophe!

(B explodes for small λ)

Spectral Blackbody: Planck's Law

- Planck's Law was found empirically (trial and error!)
- Quantize the E&M radiation so that the minimum energy for light at a given wavelength is:

$$E_\nu = h\nu = hc/\lambda$$

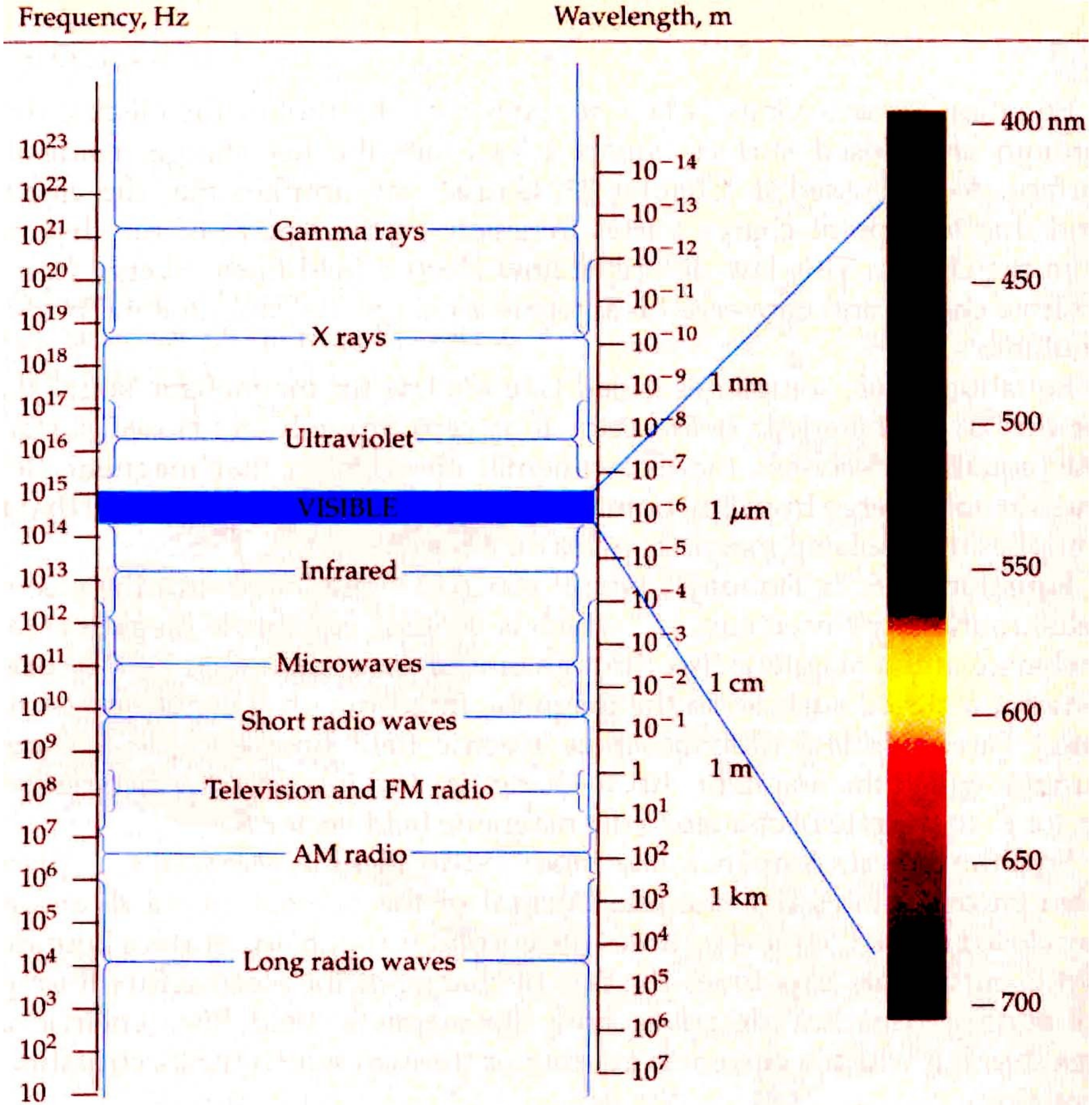
where h = Planck's Constant = 6.266 x 10^{-27} erg·s.

Then $$E_\nu = nh\nu,\ n = 0,\ 1,\ 2,\ 3$$

can be used in replacing the classical *kT* expression for the average energy in a mode.

Now the entire hot object may not have enough energy to emit one photon of light at very small wavelengths, so *n=0*, and the UV catastrophe can be avoided.

Photons: Electromagnetic Spectrum

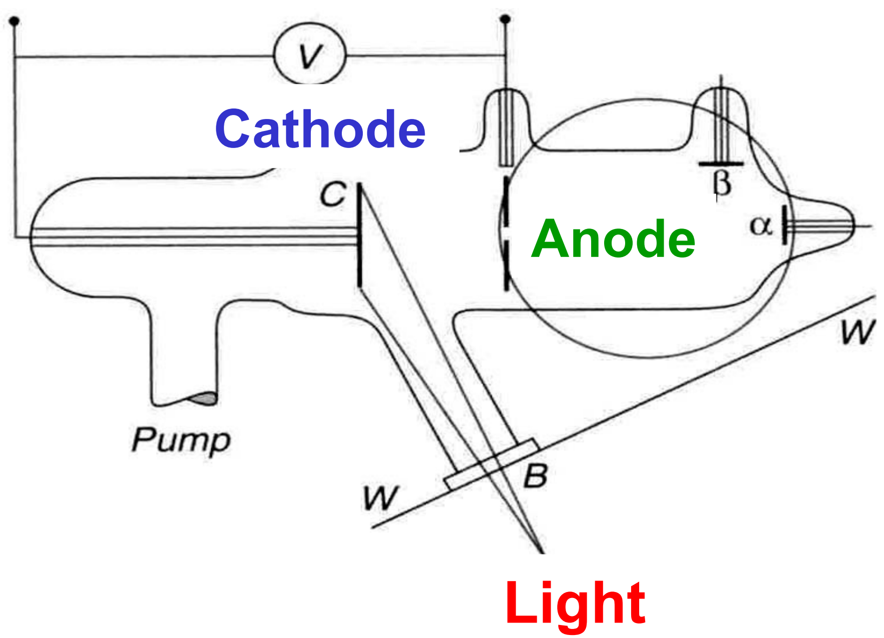


Photoelectric Effect: “Particle Behavior” of Photon

- Shows quantum nature of light (Theory by Einstein & Expt. by Millikan).

- **Photons** hit metal cathode and instantaneously eject **electrons** (requires minimum energy = work function).
- Electrons travel from cathode to anode against **retarding voltage V_R**

- Electrons collected as **“photoelectric” current** at anode.
- Photocurrent becomes zero when retarding voltage V_R equals the **stopping voltage V_{stop}**, i.e. $eV_{stop} = K_e$



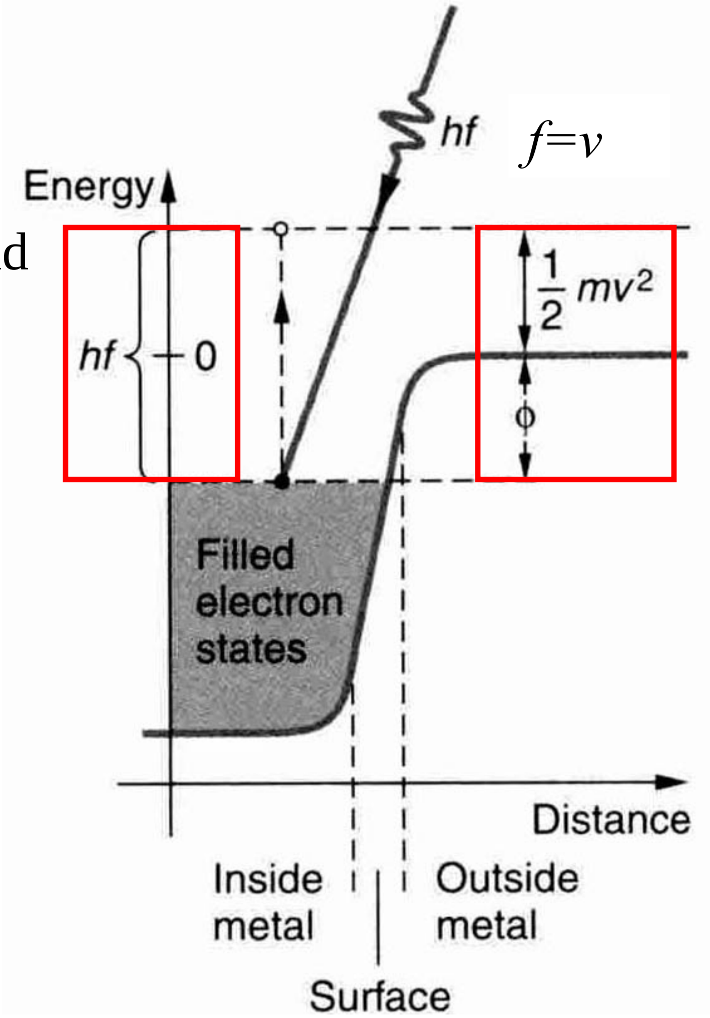
Photoelectric Effect - equation

- PHOTON IN ⇒ ELECTRON OUT
  - e- kinetic energy = Total photon energy – e- ejection energy

$$K_{\max} = hv - \varphi$$

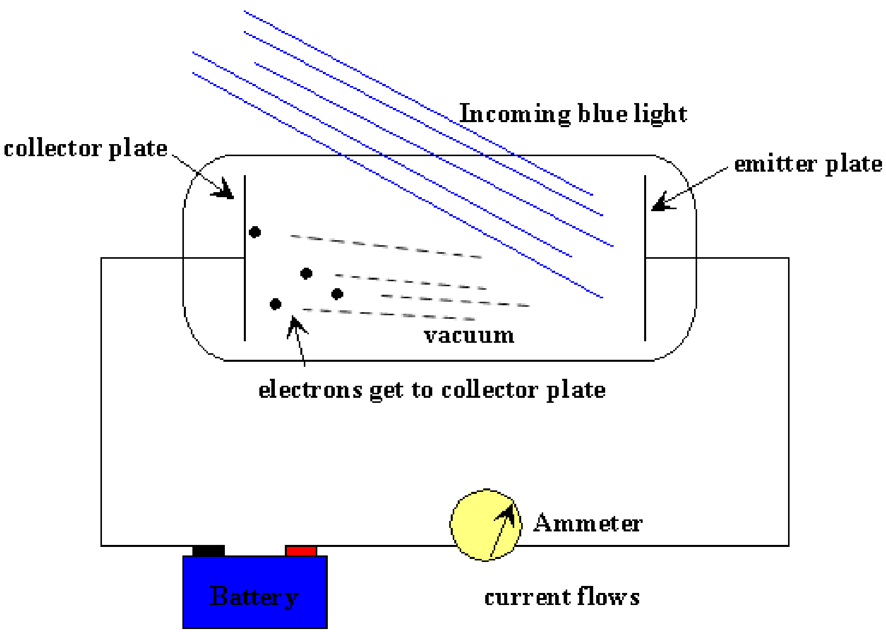
  - where hv = photon energy, ϕ = work function, and K_{max} = kinetic energy
  - $K_{max} = eV_{stop}$ = stopping energy
- Special Case: No kinetic energy ($V_o = 0$)
  - Minimum frequency v to eject electron

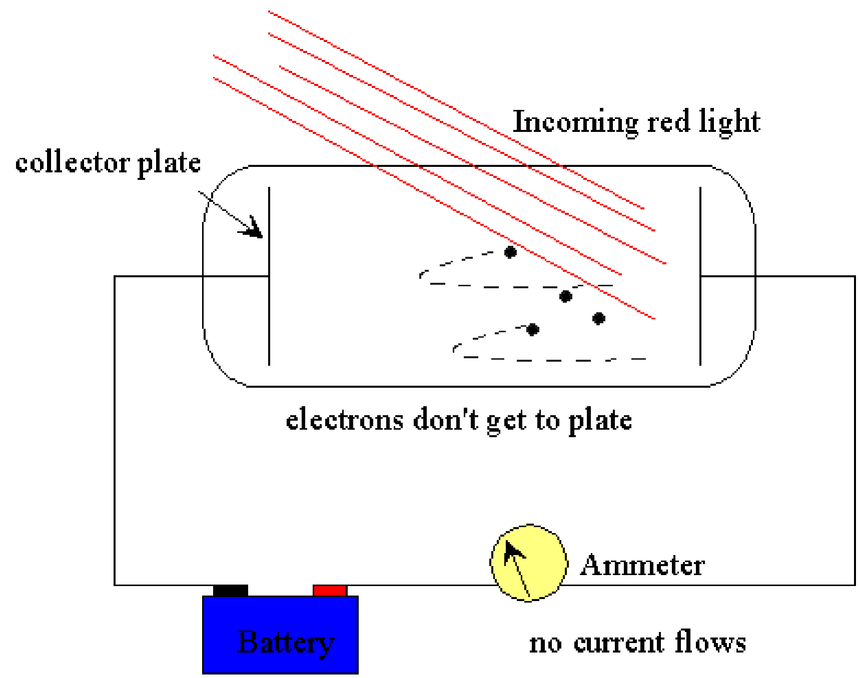
$$hv_{min} = \varphi$$



Photoelectric Effect

- In order to make electrons reach the collector plate, the light has to be “blue enough”; the intensity doesn't matter if light is red!

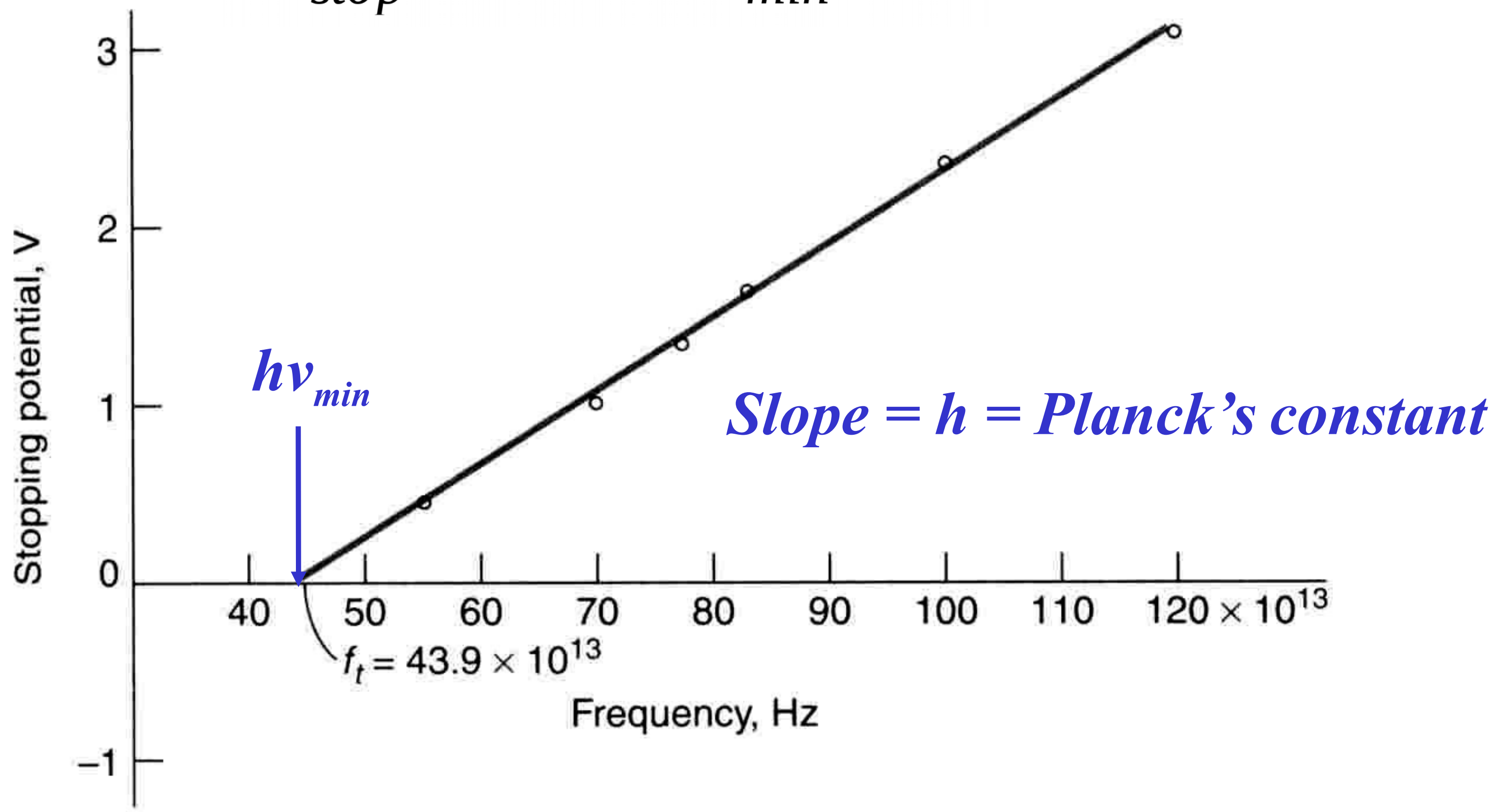




Photoelectric Effect: V_{stop} vs. Frequency

$$eV_{stop} = h\nu - \varphi$$

$$V_{stop} = 0 \Rightarrow h\nu_{min} = \varphi$$



Photoelectric Effect Problem

If the work function of a metal is 2.0 eV,

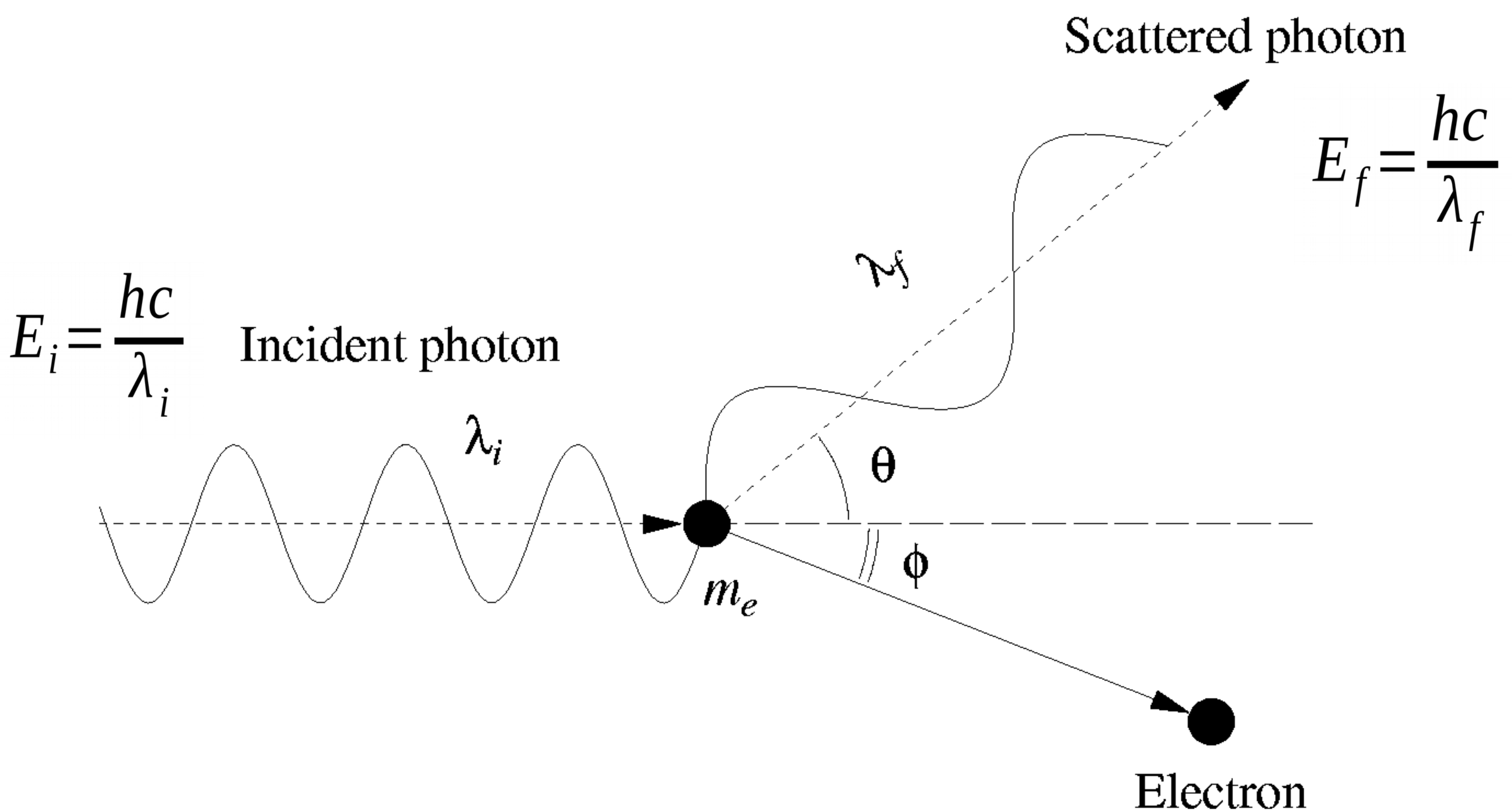
a) find the maximum wavelength λ_m capable of causing the photo-electric effect, and,

b) find the stopping potential if $\lambda = \lambda_m / 2$

Compton Scattering: "Particle-like" Behavior of Photon

Concept: Photon scatters off electron losing energy and momentum to the electron. The λ_f of scattered photon depends on θ

- Conservation of relativistic momentum and Energy!
- No mass for the photon but it has momentum!!!



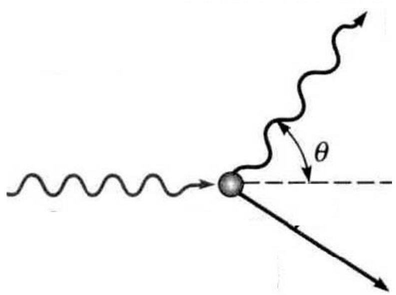
Compton Scattering: Equation

Photon OUT → λ_f; Scattering Angle → θ

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

Photon IN → λ_i

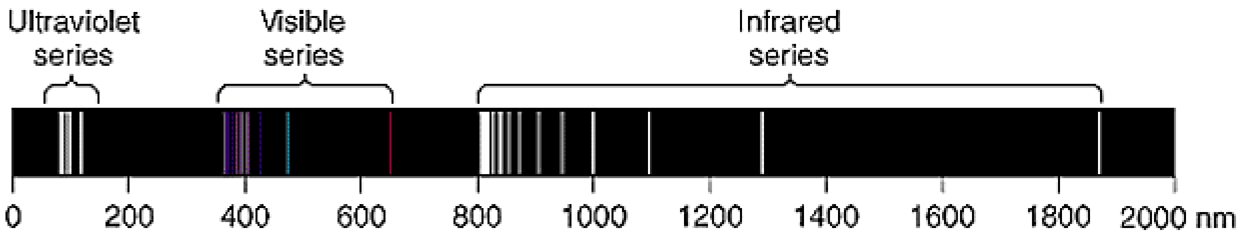
$\frac{h}{m_e c}$: Critical λ_c = 0.0024 nm for e^-



- **Limiting Values**
  - No scattering: $\theta = 0^{o} \rightarrow \cos 0^{o} = 1 \rightarrow$ **$\Delta\lambda = 0$**
  - "Bounce Back": $\theta = 180^{o} \rightarrow \cos 180^{o} = -1 \rightarrow$ **$\Delta\lambda = 2\lambda_c$**
- Difficult to observe unless λ is small (i.e. $\Delta\lambda/\lambda > 0.01$)

Atomic Spectra

- 1885 - Balmer observed Hydrogen Spectrum
  - Found empirical formula for discrete wavelengths
  - Later generalized by Rydberg for simple ionized atoms



$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \textbf{ with } 2 < n$$

Atomic Spectra: Rydberg Formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \textbf{ with } m < n$$

- Gives λ for any lower level *m* and upper level *n* of Hydrogren.
- Rydberg constant $R_H \sim 1.097 \times 10^7\ m^{-1}$
- m = 1 (Lyman), 2 (Balmer), 3 (Paschen)

- Example for **n = 2 to m = 1** transition:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} \left(1.097 \times 10^7\, m^{-1} \right)$$

$$\Rightarrow \lambda = 121.6\ nm\ Ultraviolet$$

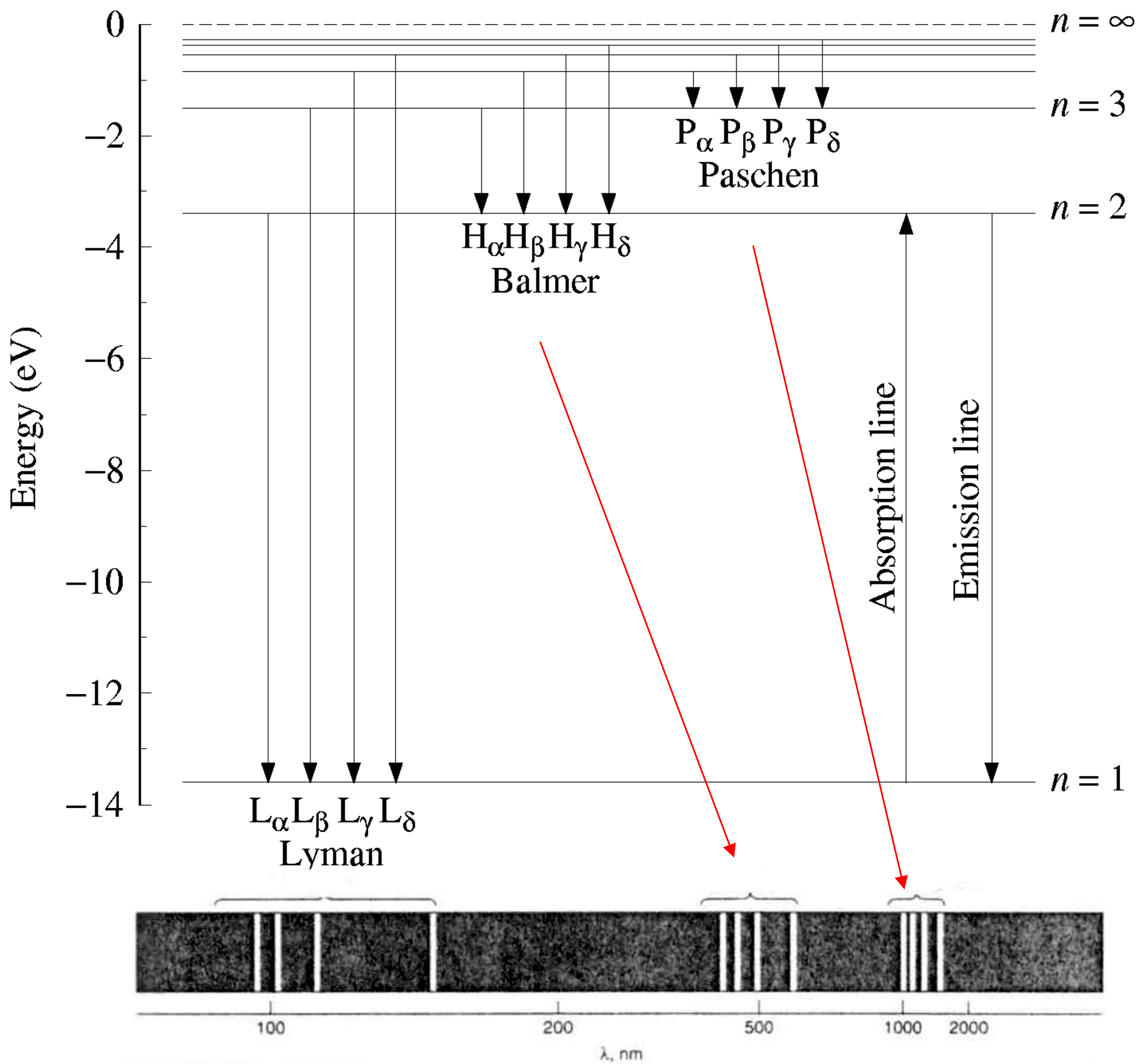
Atomic Spectra: Hydrogen Energy Levels

E_∞ = 0 eV

Energy

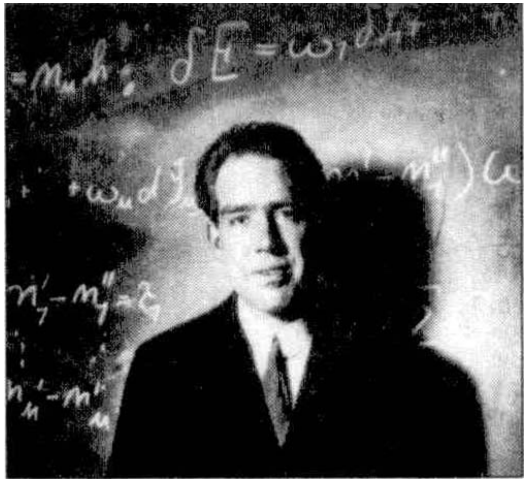
$$E_n \propto \frac{-1}{n^2}$$

E_1 = -13.6 eV



Bohr Model

- 1913 – Bohr proposed quantized model of the H atom to predict the observed spectrum.
- Problem: Classical model of the electron "orbiting" nucleus is unstable. Why unstable?
  - Electron experiences (centripetal) acceleration.
  - Accelerated electron emits radiation.
  - Radiation leads to energy loss.
  - Electron quickly "crashes" into nucleus.



Bohr Model: Quantization

- **Solution:** Bohr proposed two "quantum" postulates
  - Electrons exist in stationary orbits (no radiation) with quantized angular momentum.

$$L_n = mvr = n\hbar$$

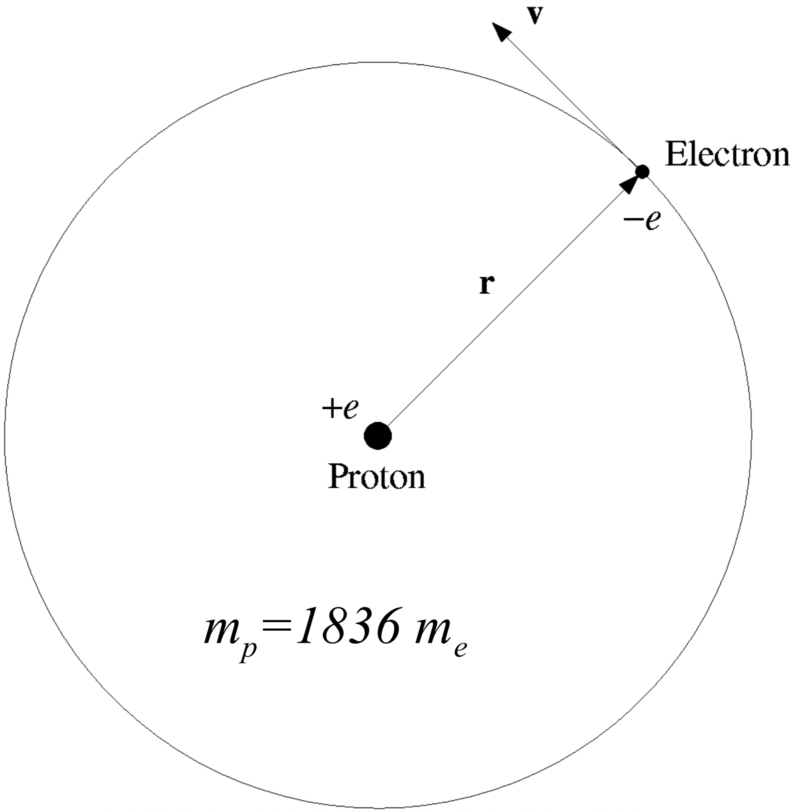
$$\left(\hbar = \frac{h}{2\pi} = 6.58\mathrm{X}10^{-16}\, eV \cdot s\right)$$

  - Atom radiates with quantized frequency ν (or energy E) only when the electron makes a transition between two stationary states.

$$h\nu = \frac{hc}{\lambda} = E_i - E_f$$

Planetary Mechanics Applied to the H Atom

- Consider the attractive electrostatic force and circular motion



$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r} = \mu \frac{v^2}{r}\hat{r}$$

Note: in cgs, $e = 4.803x10^{-10}$ esu

$$\frac{q_1 q_2}{r^2} = -\mu \frac{v^2}{r}$$

$$\frac{-e^2}{r^2} = -\mu \frac{v^2}{r}$$

$$\frac{1}{2}\mu v^2 = \frac{1}{2}\frac{e^2}{r} = K$$ Kinetic energy

$$U = -2K = -\frac{e^2}{r}$$ Potential energy

$$\mu = \frac{m_e m_p}{M} \approx m_e$$

$$M = m_e + m_p \approx m_p$$

Planetary Mechanics Applied to the H Atom

- Introduce Bohr's quantized angular momentum $L = \mu v r = n\hbar$ (wrong)

$$K = \frac{1}{2}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu r^2}$$

- Solving for r

$$r_n = \frac{\hbar^2}{\mu e^2} n^2 = a_0 n^2$$

a_0 is the Bohr radius

- Get the Total Energy in terms of n. (Recall $E_{tot}=<U>/2$)

$$E_n = -\frac{1}{2}\frac{e^2}{r} = -\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2} = \frac{-13.6\ eV}{n^2} = \frac{-E_0}{n^2}$$

- Principle quantum number, $n = 1, 2, 3, \ldots$

Bohr Model: Transitions

- Transitions predicted by Bohr yield general Rydberg formula

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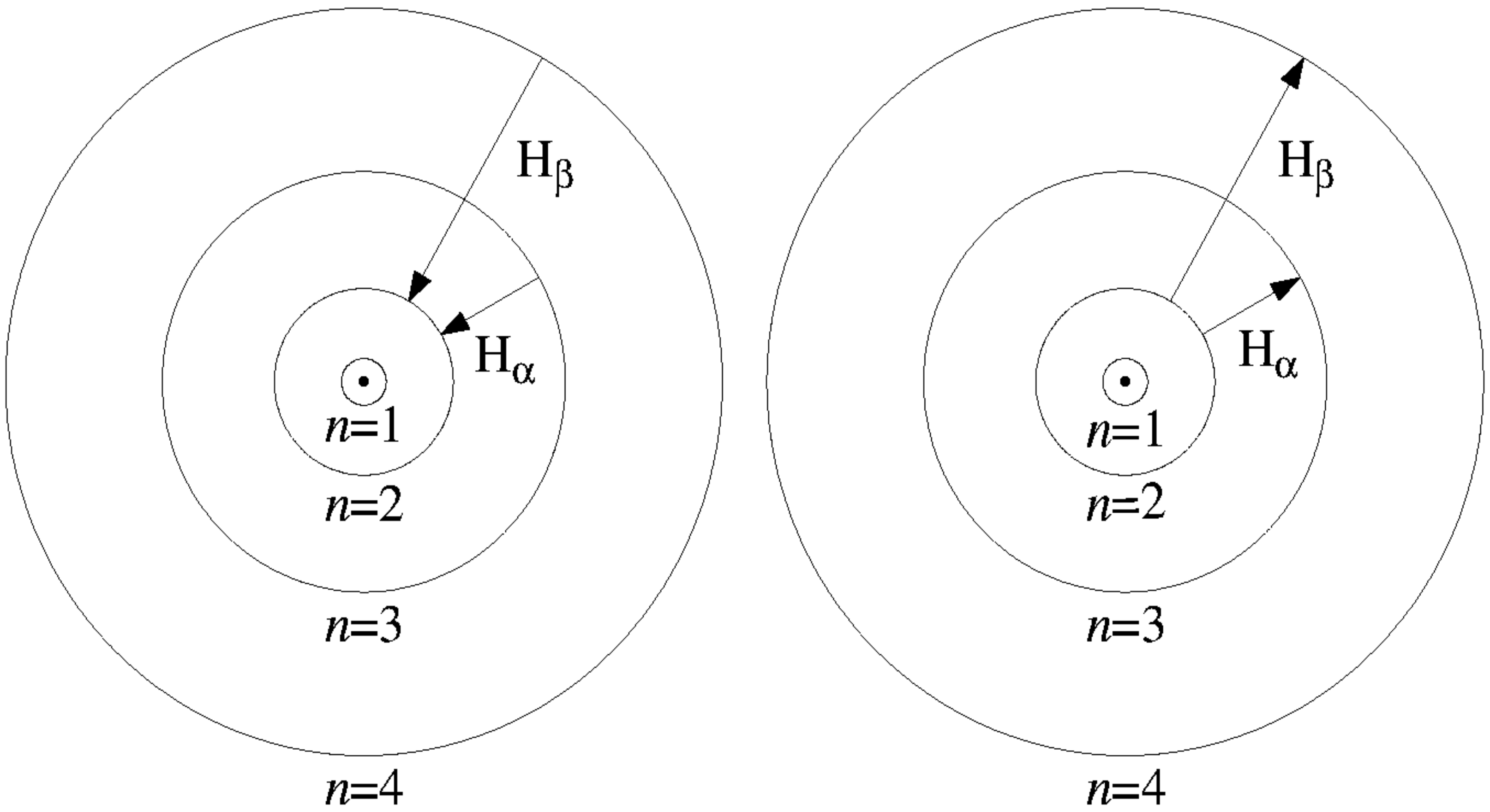

Bohr Model Problem: Unknown Transition

If the wavelength of a transition in the **Balmer series** for a **He**$^+$ atom is **121 nm**, then find the corresponding transition, i.e. initial and final n values.

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = R(2)^2\left(\frac{1}{(2)^2} - \frac{1}{n_i^{\,2}}\right)$$

where Z = 2 for He and n_f = 2 for Balmer

$$\frac{1}{4R\lambda} = \left(\frac{1}{4} - \frac{1}{n_i^{\,2}}\right)$$

$$n_i = \left(\frac{1}{4} - \frac{1}{4R\lambda}\right)^{-1/2} = \left(\frac{1}{4} - \frac{1}{4(1.1\times 10^7\,m^{-1})(121\times 10^{-9}\,m)}\right)^{-1/2} = \underline{\boldsymbol{4}}$$

Bohr Model Problem: Ionization Energy

Suppose that a He atom (Z=2) in its ground state (n = 1) absorbs a photon whose wavelength is **λ = 41.3 nm**. Will the atom be **ionized**?

- *Find the energy of the incoming photon and compare it to the ground state ionization energy of helium, or E_0 from n=1 to ∞.*

$$E = \frac{hc}{\lambda} = \frac{1240\ eV\ nm}{41.3\ nm} = \underline{\boldsymbol{30\ eV}}$$

$$E_0(He) = Z^2 \times E_0(H) = \left(2^2\right)\left(13.6\ eV\right) = 54.4\ eV$$

- *The photon energy (30 eV) is less than the ionization energy (54 eV), so the electron will NOT be ionized.*

Bohr Model Problem: Series Limit (book)

Find the **shortest wavelength** that can be emitted by the **Li $^{++}$ ion**.

➢ *The shortest λ (or highest energy) transition occurs for the highest initial state ($n_i = \infty$) to the lowest final state ($n_f = 1$).*

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

where Z = 3 for Li, $n_i = \infty$, and $n_f = 1$ for shortest λ

$$\frac{1}{\lambda} = (1.1\times 10^7\,m^{-1})(3)^2\left(\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right) = 10.1\,nm$$

Particle/Wave Duality - Part 2

PART 1

- **Electrons** as discrete **Particles**
  - Measurement of **e/m** (CRT) and **e** (oil-drop expt.)
- **Photons** as discrete **Particles**
  - **Blackbody Radiation**: Temp. Relations & Spectral Distribution
  - **Photoelectric** Effect: Photon "kicks out" Electron
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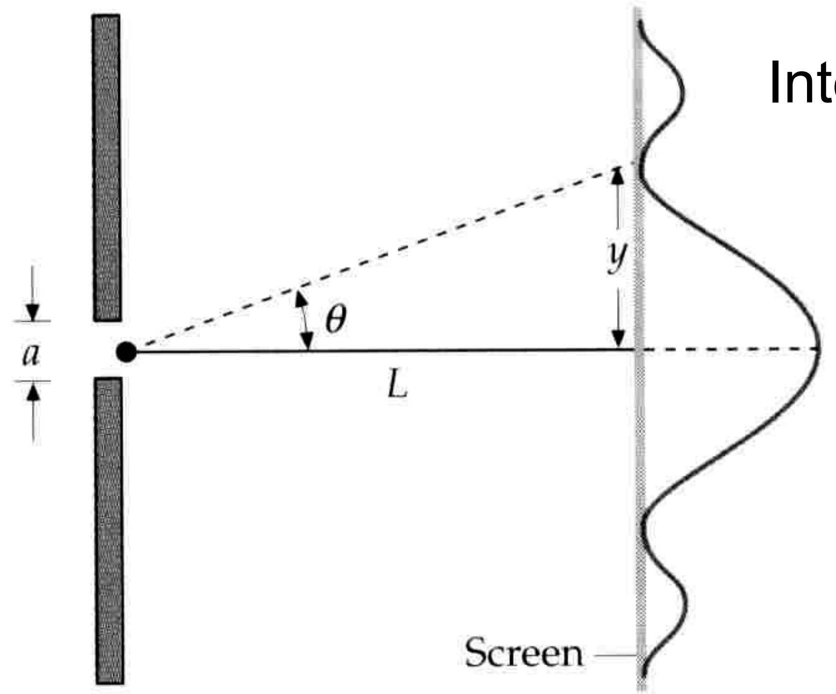
PART 2

- **Wave Behavior:** Diffraction and Interference
- **Photons** as **Waves**: $\lambda = hc / E$
  - X-ray Diffraction (Bragg's Law)
- **Electrons** as **Waves**: $\lambda = h / p$
  - Low-Energy Electron Diffraction (LEED)

Wave Property: Single-Slit Diffraction

$$\text{Minima:} \quad n\lambda = a\sin\theta$$

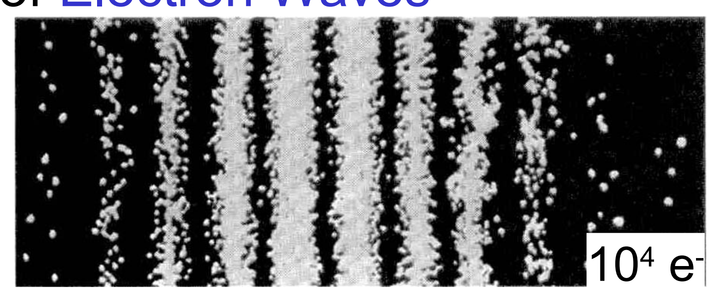
a = slit width



Intensity

$(\mathrm{Sin}(x) / x)^2$
maxima

Diffraction Pattern of Electron Waves



Electrons: Wave-like Behavior

- Every particle has a wavelength given by: $$\lambda = \frac{h}{p}$$
- **Question**: Why don't we observe effects of particle waves (i.e., diffraction and interference) in day-to-day life?
- **Answer**: Wavelengths of most macroscopic objects are too small to interact with slits, BUT atomic-sized objects DO behave like waves!

Macroscopic – ping pong ball

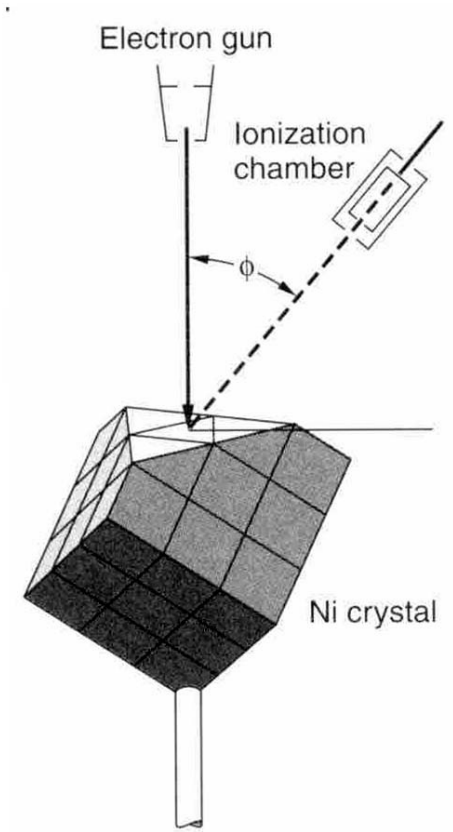
$$\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}\, J\cdot s}{\left(2\times 10^{-3} kg\right)\left(5 m/s\right)} = 6.6\times 10^{-32} m \ \left(\text{immeasurably small!}\right)$$

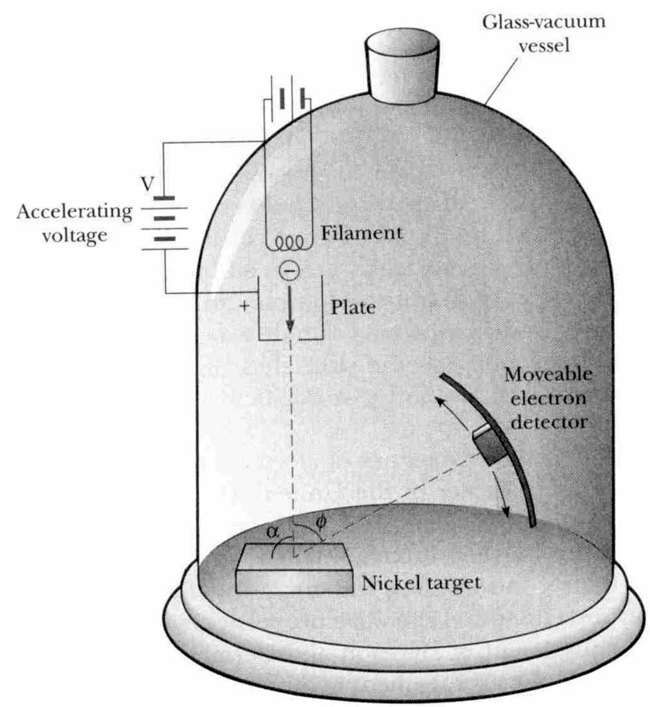
Microscopic – "slow electron" (1% speed of light)

$$\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}\, J\cdot s}{\left(9.1\times 10^{-31} kg\right)\left(10^{6} m/s\right)} = 7.3\times 10^{-10} m \left(\text{atomic dimension}\right)$$

Electron Diffraction: Wave-like Behavior

- 1927 – Davisson and Germer studied the diffraction of an electron beam from a nickel crystal surface and observed discrete spots (maxima).
- Modern day technique now: Low Energy Electron Diffraction **(LEED).**

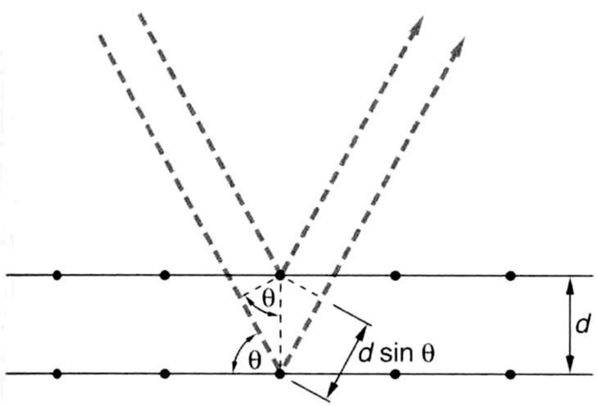




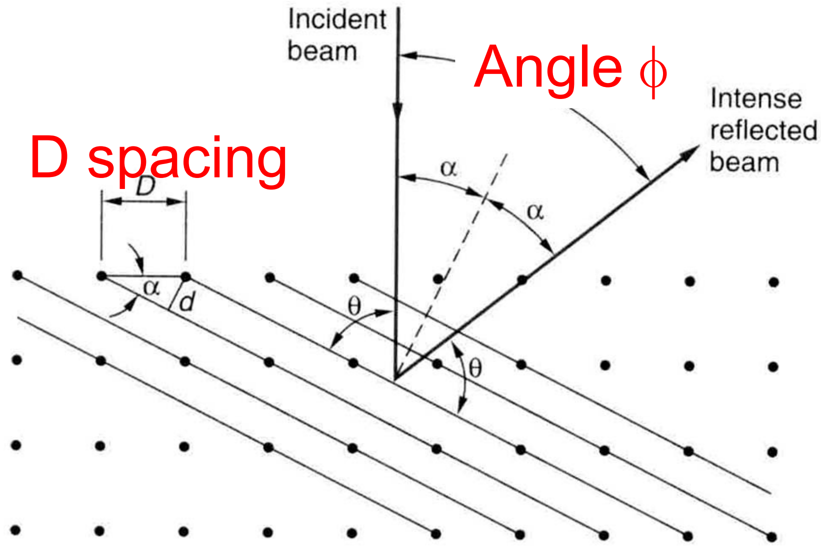
Electron Diffraction: LEED Equation

Concept: Use Bragg's Law for X-ray scattering and then substitute appropriate angles, where λ is now the electron wavelength.

X-ray Diffraction



Electron Diffraction



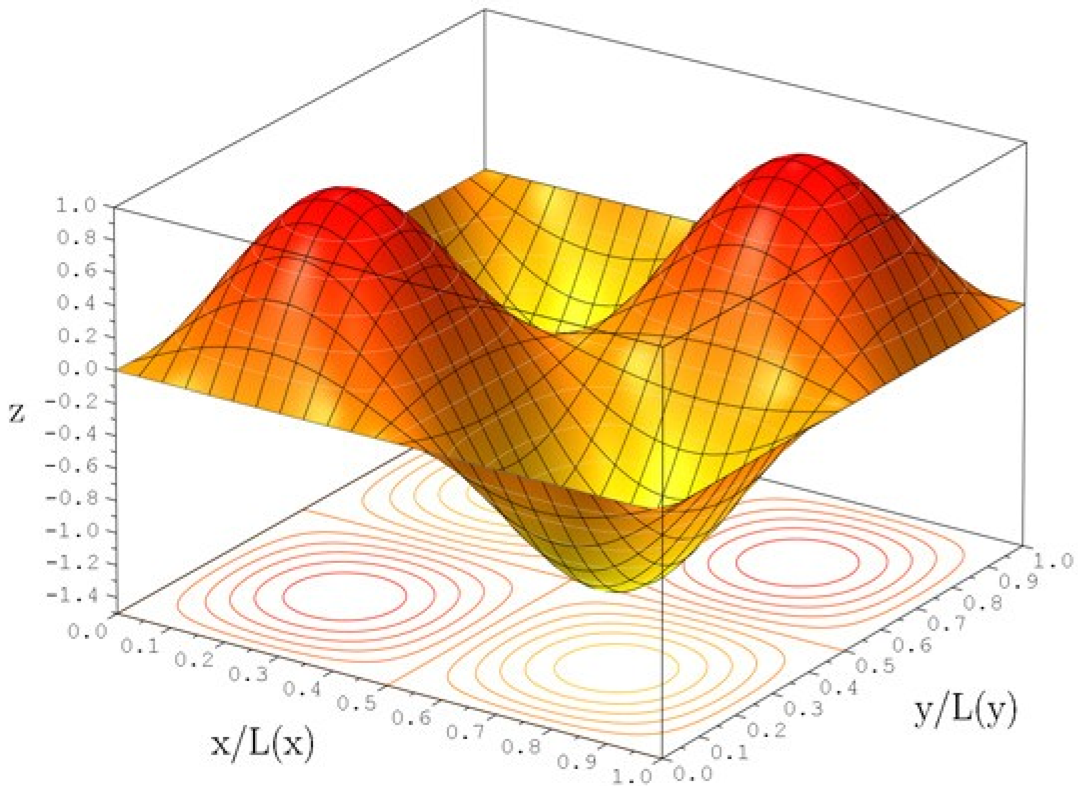
$$n\lambda = 2\,\underbrace{d}_{D\sin\alpha}\,\underbrace{\sin\theta}_{\cos\alpha} = 2D\,\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha \text{ by trig}} = D\sin 2\alpha$$

Dsinα cosα ½sin2α by trig

$$n\lambda = D\sin 2\alpha = D\sin\varphi$$

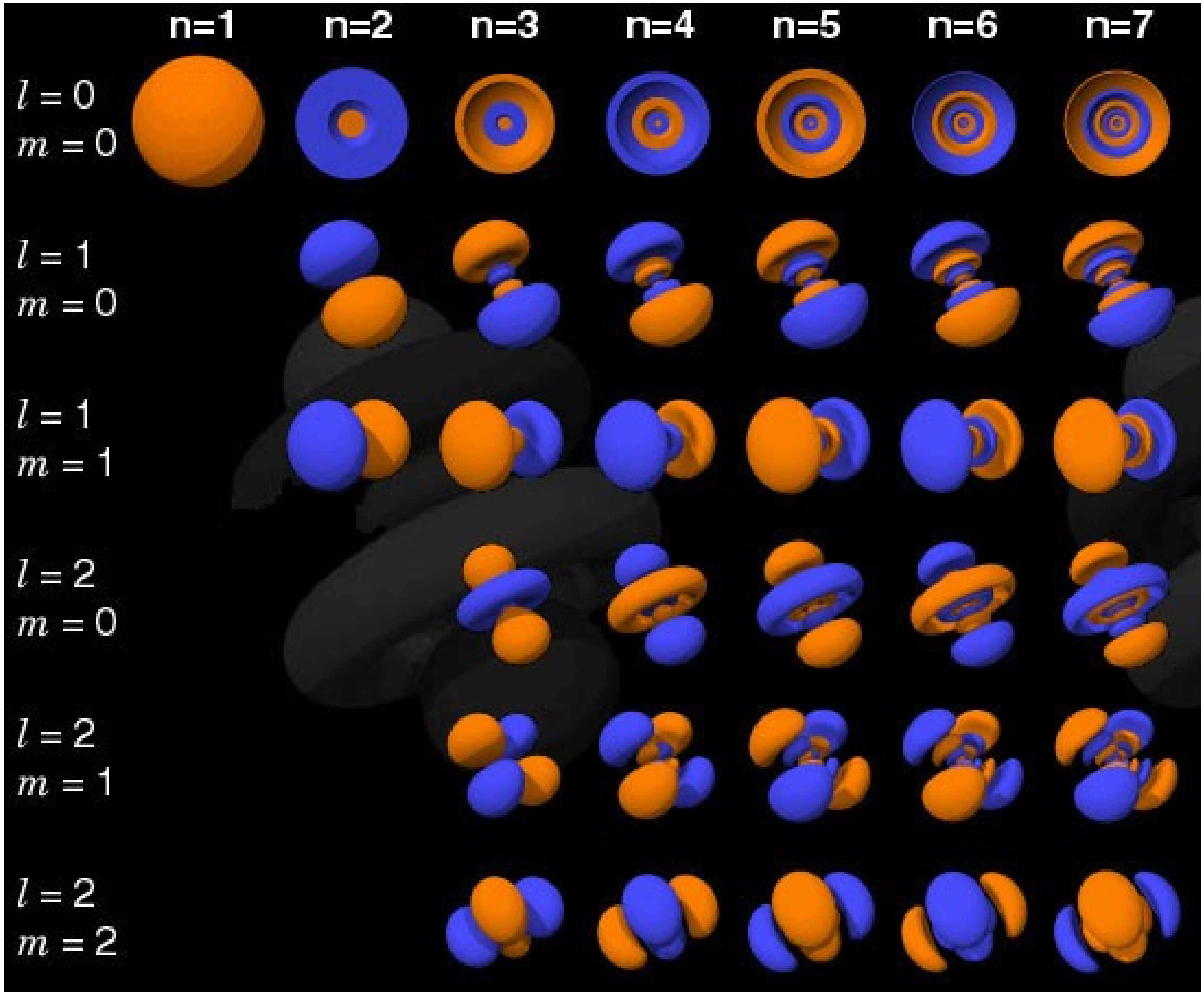
Wave/Particle Duality

- The particle wavefunction, ψ, is the "probability amplitude" (see figure "Z"), a complex number.
- Probability density = $|\Psi|^2$ gives the probability of where we might find the particle. (this must be positive)
- Can have destructive and constructive interference



Wave/Particle Duality

- This picture shows some of the possible electron probability densities for different quantum states of the H atom.
- Electron "clouds"



- Probability "clouds"
  - kind of the opposite of the "Plum Pudding" model