Example of error propagation: find the specific heat of copper

Actual student data:

$$\begin{array}{lll} m_{Al} = 61 \pm 0.5 \text{ g} & T_{equil} = 34.5 \pm 0.5°\text{C} & c_{H_2O} = 1.000 \text{ cal/(g °C)} \\ m_{Cu} = 200 \pm 0.5 \text{ g} & T_1 = 23 \pm 0.5°\text{C} & c_{Al} = 0.215 \text{ cal/(g °C)} \\ m_{wateradded} = 105 \pm 0.5\text{g} & T_2 = 96 \pm 0.5°\text{C} & \end{array}$$

Derived formula for specific heat of copper:

$$c_{Cu} = \frac{c_{Al} m_{Al}(T_{equil} - T_1) + c_{watr} m_{watr}(T_{equil} - T_1)}{m_{Cu}(T_2 - T_{equil})}$$

Break up formula into easier chunks:

$$c_{Cu} = Q = \frac{A + B}{C}$$

where

$$A = (0.215)(61)(11.5) = 150.82 \text{ cal}$$

$$B = (1.0)(105)(11.5) = 1207.5 \text{ cal}$$

$$C = (200)(61.5) = 12300 g°C$$

Find errors for A, B, and C using propagation formula # 2:

$$\frac{\delta A}{A} = \sqrt{\left(\frac{0}{.215}\right)^2 + \left(\frac{.5}{61}\right)^2 + \left(\frac{.7}{11.5}\right)^2}$$

$$\delta A = A\sqrt{0 + 6.7 \times 10^{-5} + .0037} = 9.4\text{cal}$$

$$\frac{\delta B}{B} = \sqrt{\left(\frac{0}{1}\right)^2 + \left(\frac{.5}{105}\right)^2 + \left(\frac{.7}{11.5}\right)^2}$$

$$\delta B = B\sqrt{0 + 2.3 \times 10^{-5} + .0037} = 73.7\text{cal}$$

$$\frac{\delta C}{C} = \sqrt{\left(\frac{0.5}{200}\right)^2 + \left(\frac{.7}{61.5}\right)^2}$$

$$\delta C = C\sqrt{6.3 \times 10^{-5} + 1.3 \times 10^{-4}} = 170.7\text{g °C}$$

Find error for numerator, $A + B$, using propagation formula #1:

$$A + B = 1358.3\text{cal}$$

$$\delta(A + B) = \sqrt{\delta A^2 + \delta B^2} = \sqrt{88.4 + 5432} = 74.3\text{cal}$$

Finally, find the error for Q using propagation formula #2 again:

$$Q = C_{Cu} = \frac{1358.3}{12300} = 0.110\text{cal/(g °C)}$$

$$\frac{\delta Q}{Q} = \sqrt{\left(\frac{74.3}{1358.3}\right)^2 + \left(\frac{170.7}{12300}\right)^2}$$

$$\delta Q = Q\sqrt{3 \times 10^{-3} + 1.9 \times 10^{-4}} = 0.110(0.0565)$$

$$\delta Q = 0.006\text{cal/(g °C)}$$

Thus, the measurement for the specific heat of copper is:

$$c_{Cu} = 0.110 \pm 0.006\text{cal/(g °C)}$$