Chapter 30

Sources of the Magnetic Field

PHYS 2321
Week 10: Sources of Magnetic field

Day 1 Outline

1) Hwk: Ch. 28 P. 1,4,5,19,27,29,37,38 Due Wed → Fri
   MiscQ 1-13 (odd)
2) Quiz on Ch. 27 – magnetic fields and forces
3) Applications of crossed E and B fields
4) Sources of magnetic fields (Ch 28)
   a. Demo: current makes B-field
   b. Biot-Savart law
   c. B of a long straight wire
   d. Force between parallel currents

Notes:

PHYS 2321
Week 10: Sources of Magnetic field

Day 2 Outline

1) Hwk: Ch. 28 P. 1,4,5,19,27,29,37,38 Due Fri
   MiscQ 1-13 (odd)

2) Sources of magnetic fields (Ch 28)
   a. Demo: current makes B-field ✓
   b. Biot-Savart law
   c. B of a long straight wire
   d. Force between parallel currents
   e. Ampere's law – easier than Biot-Savart!

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$

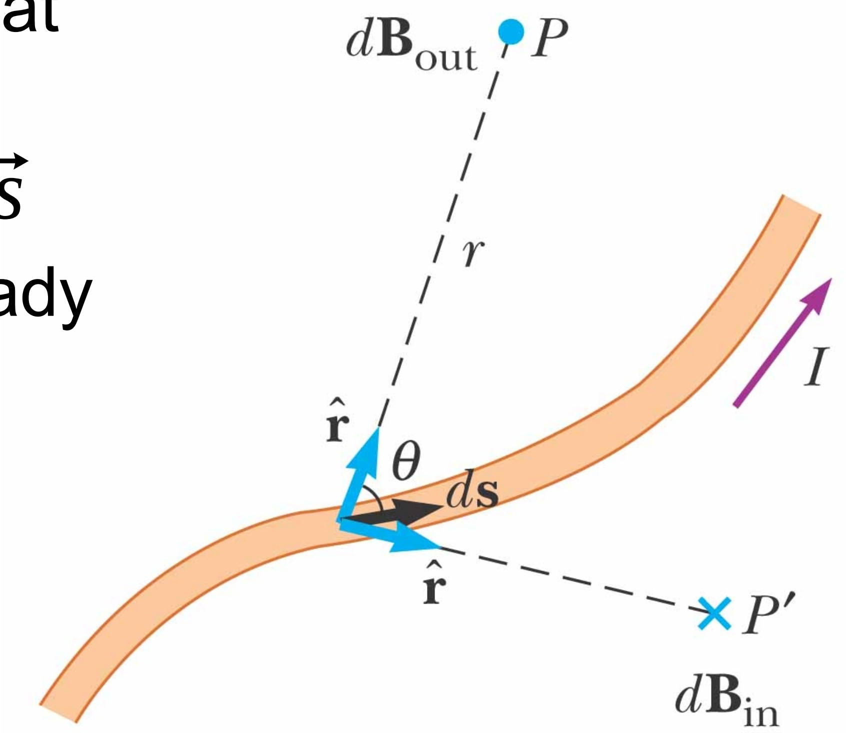
Notes:

Biot-Savart Law – Introduction

- Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet
- They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current

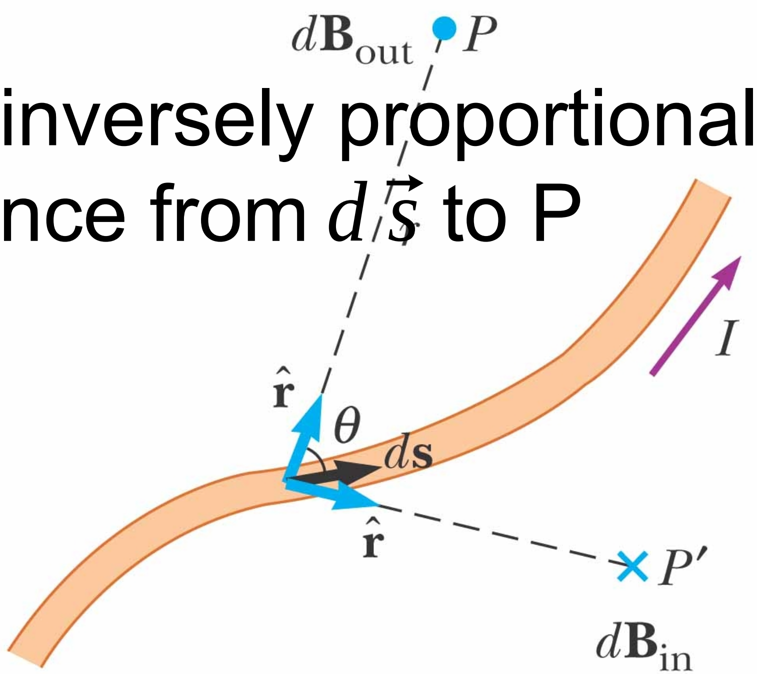
Biot-Savart Law – Set-Up

- The magnetic field is $d\vec{B}$ at some point *P*.
- The length element is $d\vec{s}$
- The wire is carrying a steady current, I



Biot-Savart Law – Observations

- The vector $d\vec{B}$ is perpendicular to both $d\vec{s}$ and to the unit vector $\hat{r}$ directed from $d\vec{s}$ toward P
- The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{s}$ to P



Biot-Savart Law – Observations, cont

- The magnitude of $d\vec{B}$ is proportional to the current and to the magnitude *ds* of the length element $d\vec{s}$
- The magnitude of $d\vec{B}$ is proportional to sin θ, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$

Biot-Savart Law – Equation

- The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$

- The magnetic field described by the law is the field *due to* the current-carrying segment $d\vec{s}$.
  - It doesn't include the B due to other currents or permanent magnets.

Permeability of Free Space

- The constant μ_o is called the **permeability of free space**
- $\mu_o = 4\pi \times 10^{-7}$ T· m / A

Total Magnetic Field

- $d\vec{B}$ is the field created by the current in the length segment ds
- To find the total field, sum up the contributions from all the current elements I$d\vec{s}$

$$\vec{B}_{tot} = \frac{\mu_0 I}{4\pi} \int_{current} \frac{d\vec{s} \times \hat{r}}{r^2}$$

  - The integral is over the entire current distribution

$\vec{B}$ Compared to $\vec{E}$

- Distance
  - The magnitude of the magnetic field varies as the inverse square of the distance from the source
  - The electric field due to a point charge also varies as the inverse square of the distance from the charge

$\vec{B}$ Compared to $\vec{E}$, 2

- Direction
  - The electric field created by a point charge is radial in direction
  - The magnetic field created by a current element is perpendicular to both the length element $d\vec{s}$ and the unit vector $\hat{r}$

$\vec{B}$ Compared to $\vec{E}$, 3

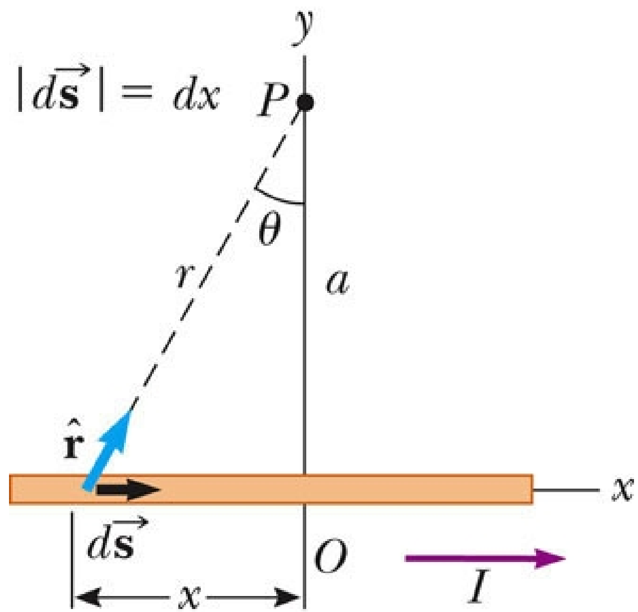
- Source
  - An electric field is established by an isolated electric charge
  - The current element that produces a magnetic field must be part of an extended current distribution
    - Therefore you must integrate over the entire current distribution

$\vec{B}$ for a Long, Straight Conductor

- The thin, straight wire is carrying a constant current, I
- $d\vec{s}\times\hat{r}=\left(dx\sin\left(90-\theta\right)\right)\hat{k}$
- Integrating over all the current elements gives

$$B=-\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta d\theta$$

$$B=\frac{\mu_0 I}{4\pi a}\left(\sin\theta_1-\sin\theta_2\right)$$



(a)

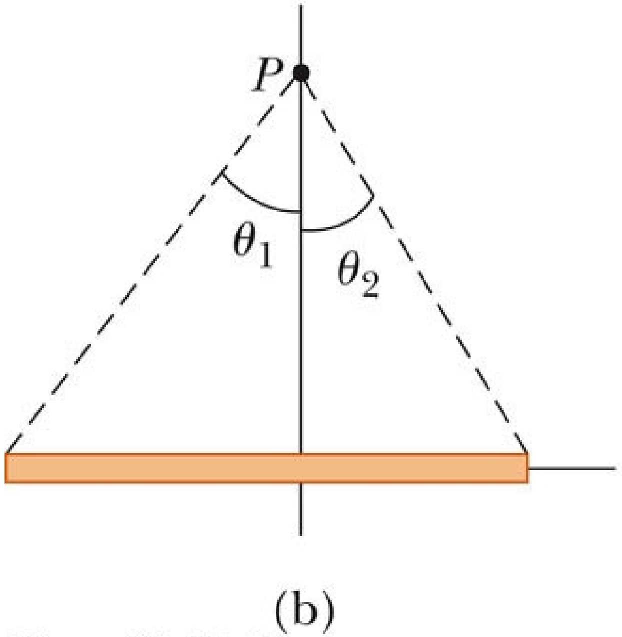
$\vec{B}$ for a Long, Straight Conductor, Special Case

- If the conductor is an infinitely long, straight wire, $\theta_1 = \pi/2$ and

  $\theta_2 = -\pi/2$

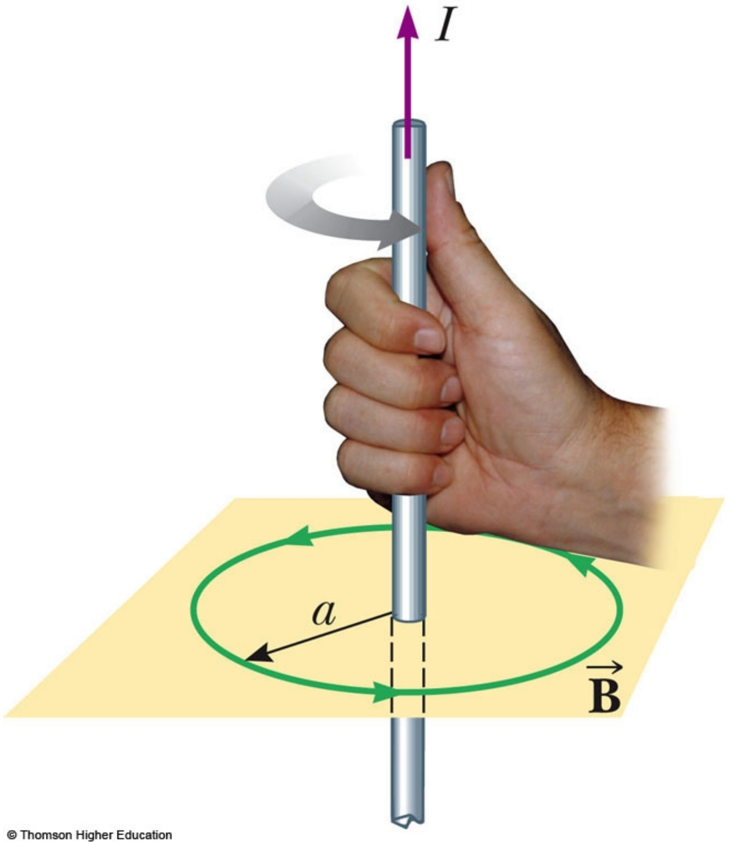
- The field becomes

$$B = \frac{\mu_0 I}{2\pi r}$$



B for a Long, Straight Conductor, Direction

- The magnetic field lines are circles concentric with the wire
- The field lines lie in planes perpendicular to to wire
- The right-hand rule for determining the direction of the field is shown

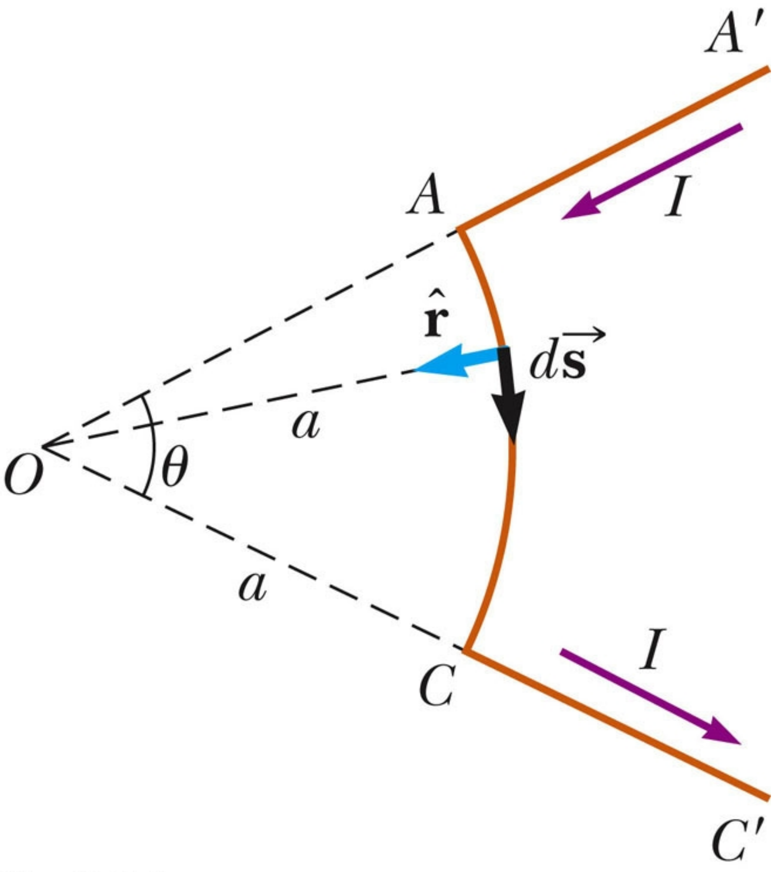


B for a Curved Wire Segment

- Find the field at point *O* due to the wire segment
- *I* and *R* are constants

  $B = \mu_0 I\theta / 4\pi a$

  θ will be in radians



B for a Circular Loop of Wire

- Consider the previous result, with a full circle
  - $\theta = 2\pi$

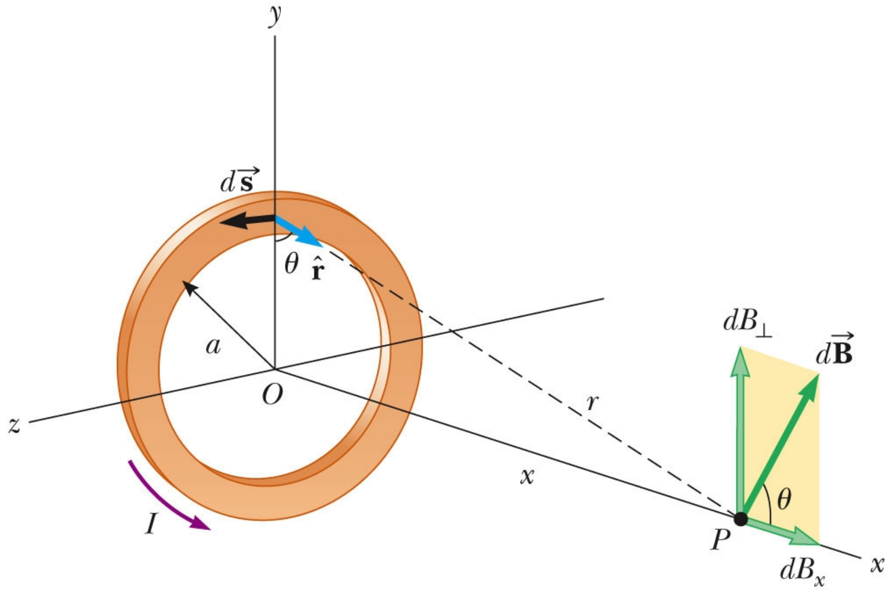
$$B = \frac{\mu_0 I\theta}{4\pi a} = \frac{\mu_0 I 2\pi}{4\pi a} = \frac{\mu_0 I}{2a}$$

- This is the field at the *center* of the loop

B for a Circular Current Loop

- The loop has a radius of R and carries a steady current of I
- Find the field at point P

$$B_x = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}}$$



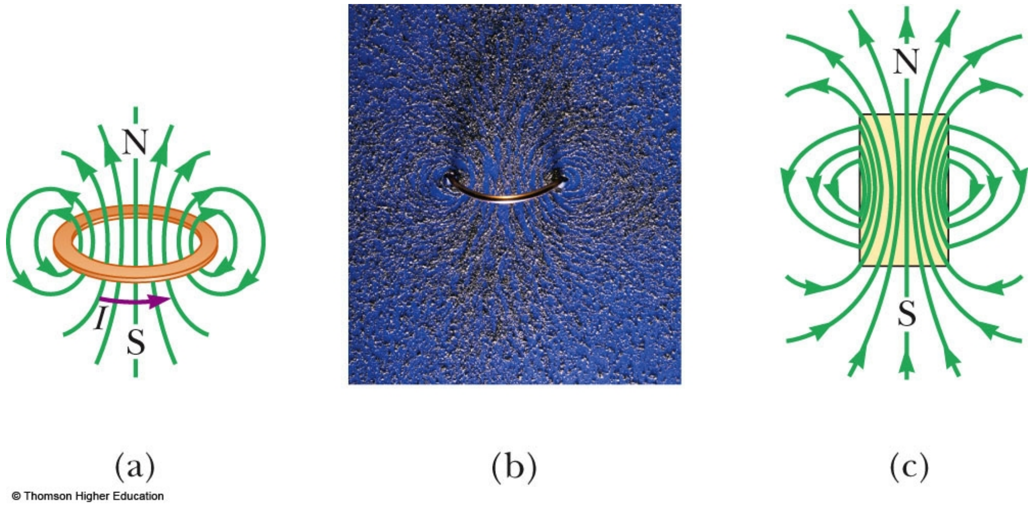
Comparison of Loops

- Consider the field at the center of the current loop
- At this special point, $x = 0$
- Then,

$$B_x = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_0 I}{2a}$$

  - This is exactly the same result as from the curved wire

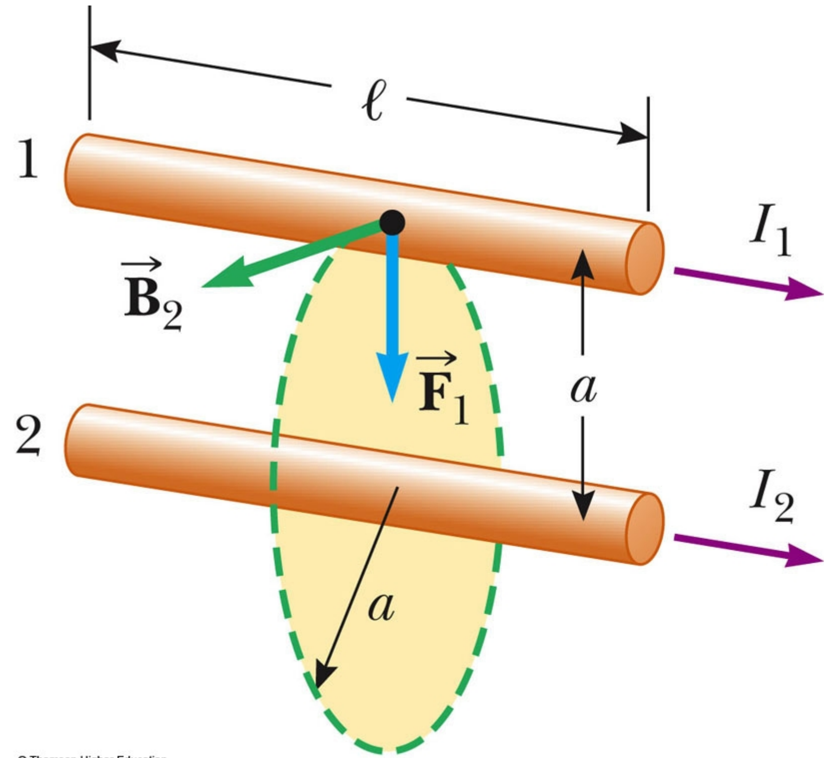
Magnetic Field Lines for a Loop



- Figure (a) shows the magnetic field lines surrounding a current loop
- Figure (b) shows the field lines in the iron filings
- Figure (c) compares the field lines to that of a bar magnet

Magnetic Force Between Two Parallel Conductors

- Two parallel wires each carry a steady current
- The field $\vec{B}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$



PLAY ACTIVE FIGURE

Magnetic Force Between Two Parallel Conductors, cont.

- Substituting $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi a}$ gives

$$F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi a}$$

  - RHR #1 shows us F_1 is down and F_2 is up.
  - Parallel conductors carrying currents in the same direction attract each other
  - Parallel conductors carrying current in opposite directions repel each other

Magnetic Force Between Two Parallel Conductors, final

- The result is often expressed as the magnetic force between the two wires, F_B
- This can also be given as the force per unit length:

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Definition of the Ampere

- The force between two parallel wires can be used to define the ampere
- When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A

Definition of the Coulomb

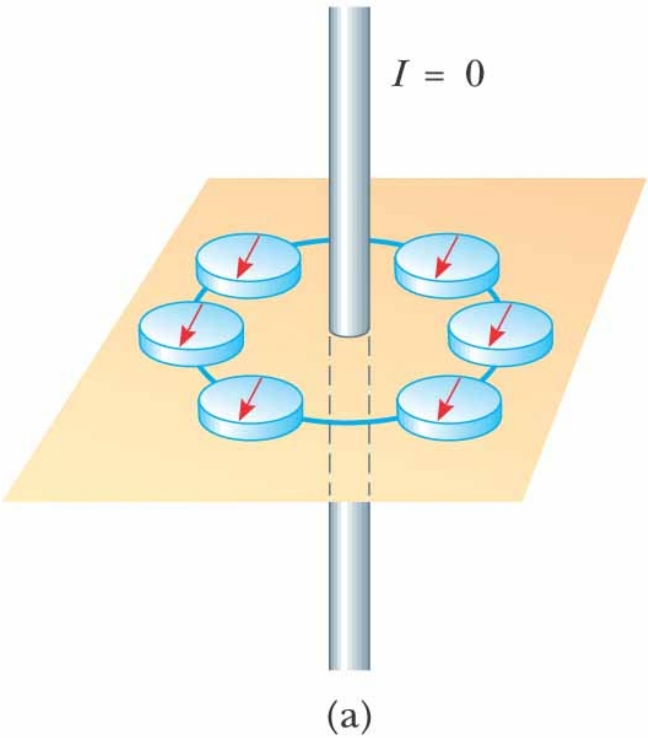
- The SI unit of charge, the coulomb, is defined in terms of the ampere
- When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C

Andre-Marie Ampère

- 1775 – 1836
- French physicist
- Credited with the discovery of electromagnetism
  - The relationship between electric current and magnetic fields
- Also worked in mathematics

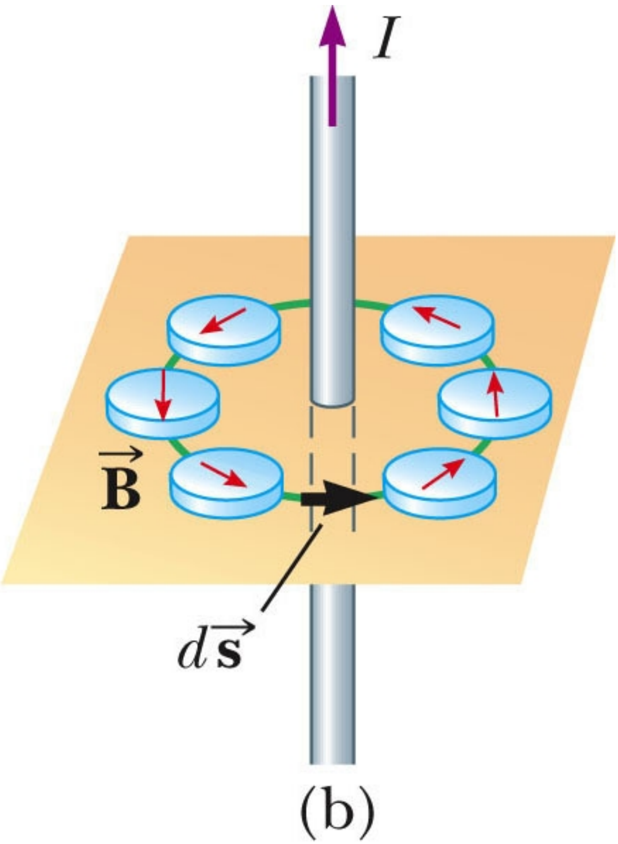
Magnetic Field of a Wire

- A compass can be used to detect the magnetic field
- When there is no current in the wire, there is no field due to the current
- The compass needles all point toward the Earth's north pole
  - Due to the Earth's magnetic field



Magnetic Field of a Wire, 2

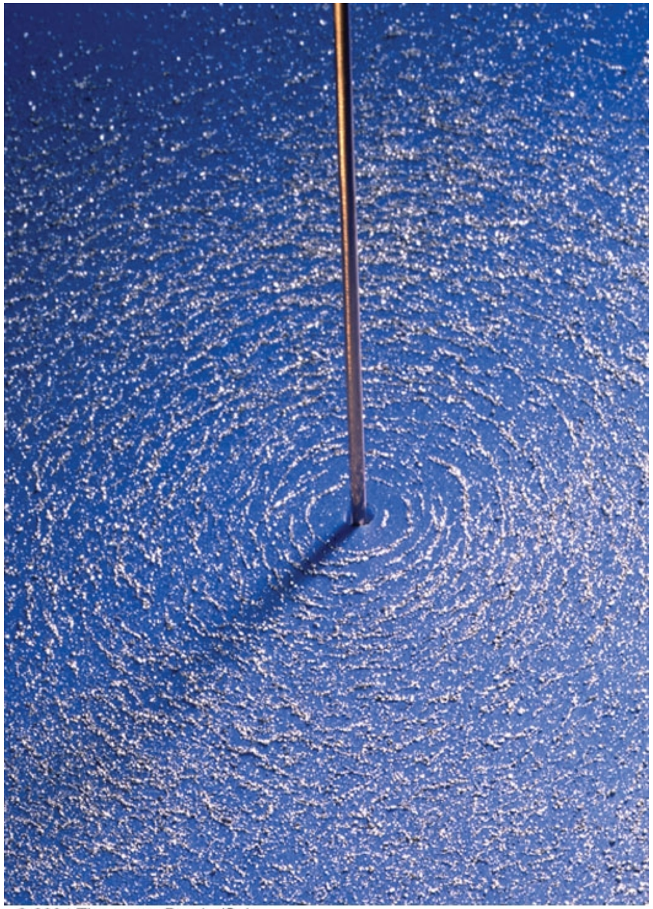
- Here the wire carries a strong current
- The compass needles deflect in a direction tangent to the circle
- This shows the direction of the magnetic field produced by the wire
- Use the active figure to vary the current



PLAY ACTIVE FIGURE

Magnetic Field of a Wire, 3

- The circular magnetic field around the wire is shown by the iron filings

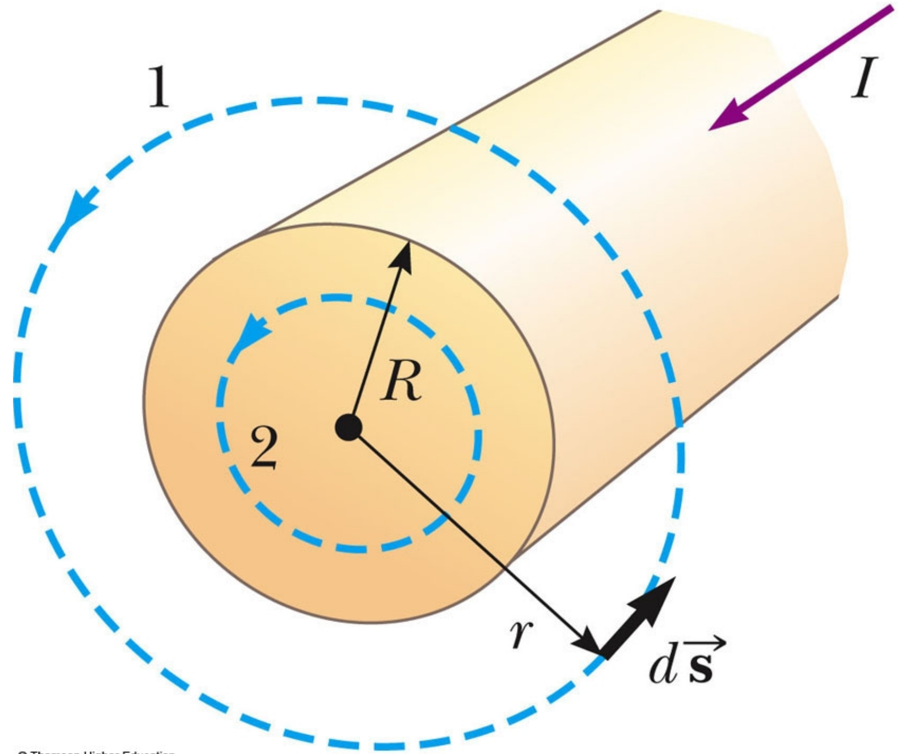


Ampere's Law

- The product of $\vec{B} \cdot d\vec{s}$ can be evaluated for small length elements $d\vec{s}$ on the circular path defined by the compass needles for the long straight wire
- Ampere's law states that the line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_o I_{enc}$ where I_{enc} is the total steady current passing through any surface bounded by the closed path: $\vec{B} \cdot d\vec{s}$

Field Due to a Long Straight Wire – From Ampere's Law

- Want to calculate the magnetic field at a distance *r* from the center of a wire carrying a steady current I
- The current is uniformly distributed through the cross section of the wire



Field Due to a Long Straight Wire – Results From Ampere's Law

- Outside of the wire, *r* > *R*

$$\int \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 I$$

$$\int \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 I$$

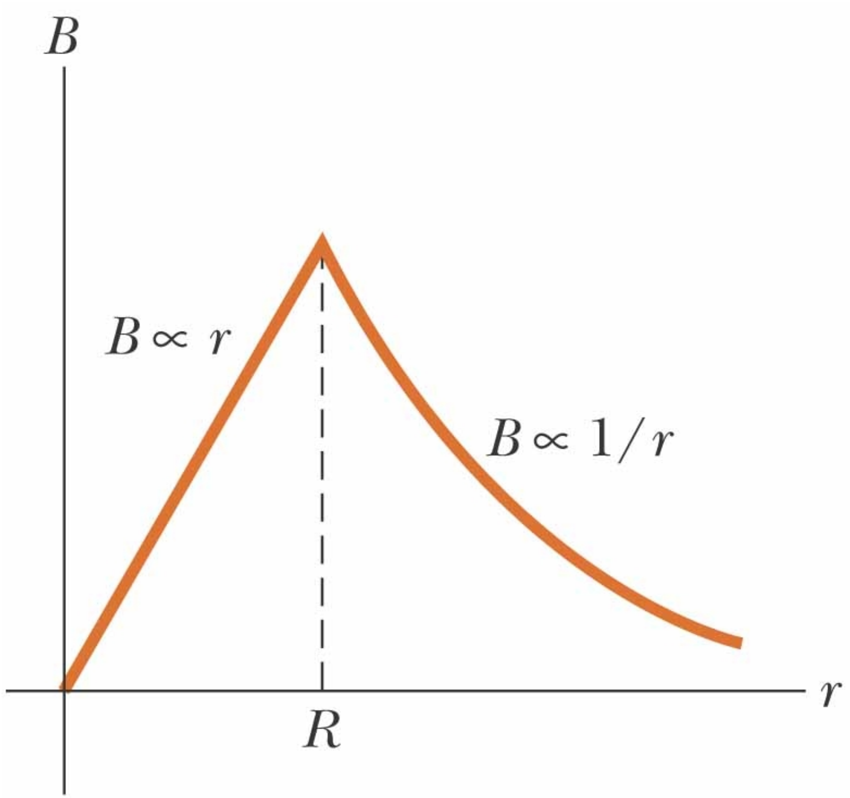
- Inside the wire, we need *I'*, the current inside the amperian circle

$$\int \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 I' \leftarrow I' = \frac{r^2}{R^2} I$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Field Due to a Long Straight Wire – Results Summary

- The field is proportional to *r* inside the wire
- The field varies as 1/*r* outside the wire
- Both equations are equal at *r* = *R*

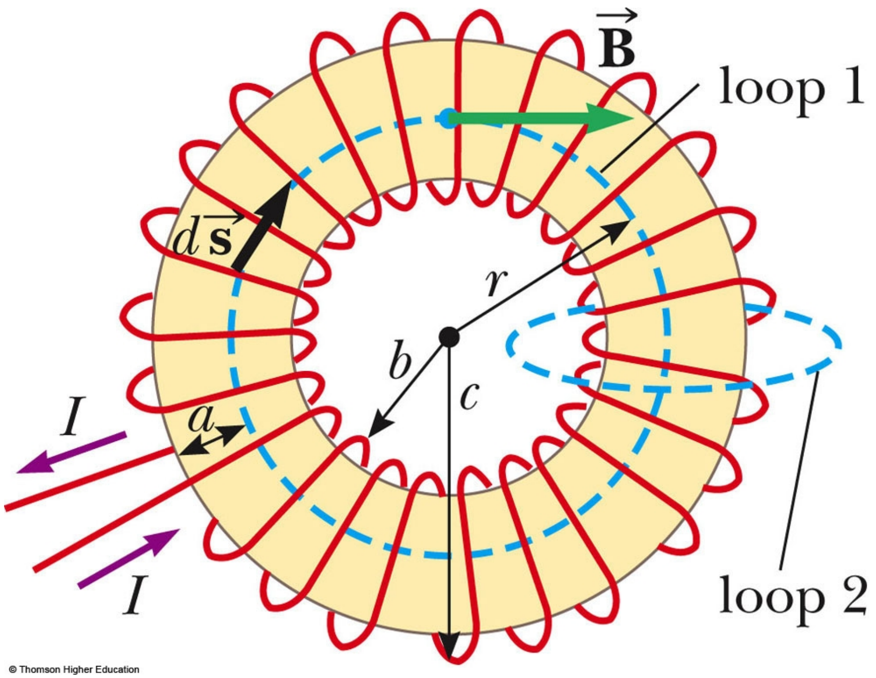


Magnetic Field of a Toroid

- Find the field at a point at distance *r* from the center of the toroid
- The toroid has *N* turns of wire

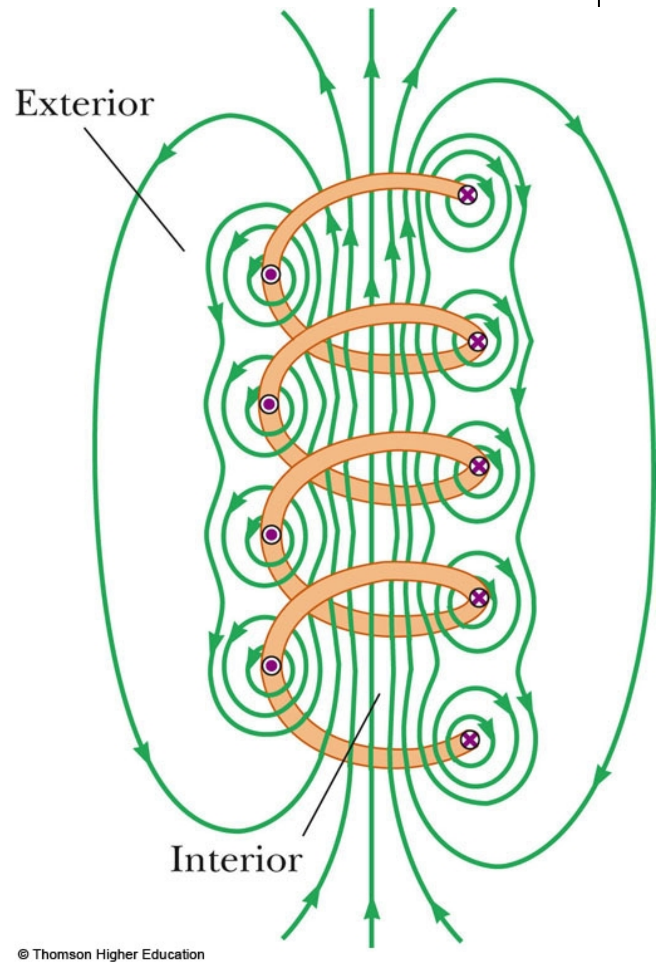
$$\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$



Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix
- A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire

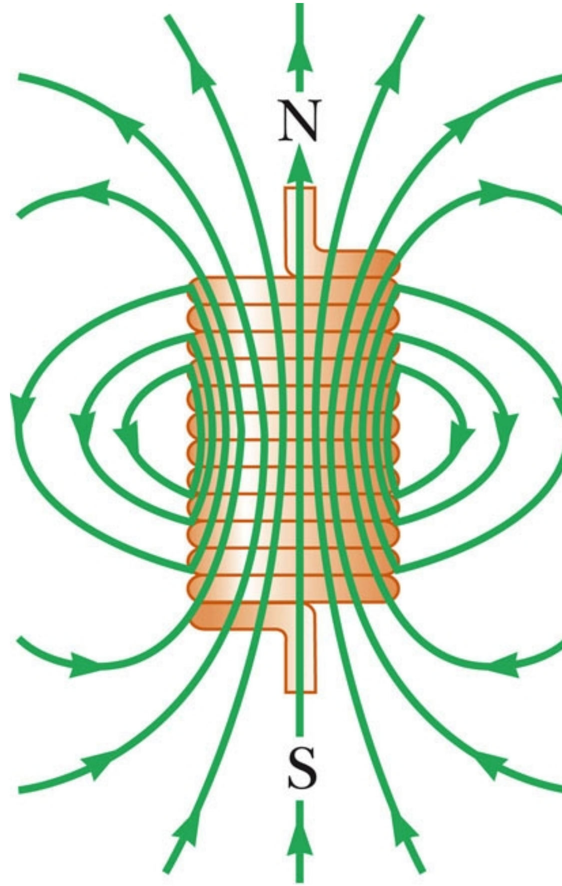


Magnetic Field of a Solenoid, Description

- The field lines in the interior are
  - nearly parallel to each other
  - uniformly distributed
  - close together
- This indicates the field is strong and almost uniform

Magnetic Field of a Tightly Wound Solenoid

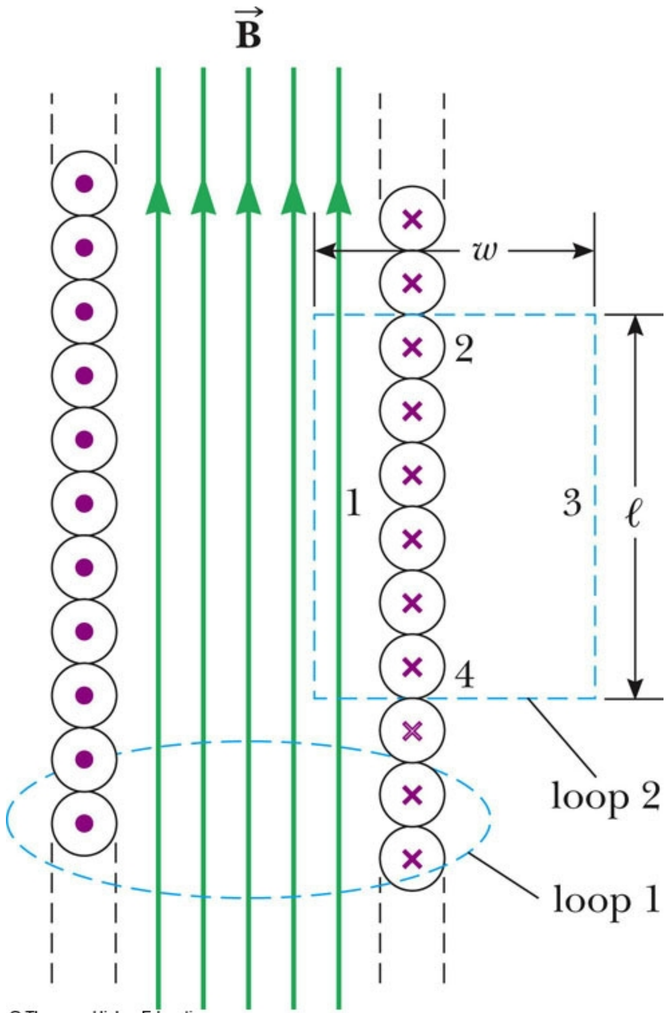
- The field distribution is similar to that of a bar magnet
- As the length of the solenoid increases
  - the interior field becomes more uniform
  - the exterior field becomes weaker



(a)

Ideal Solenoid – Characteristics

- An *ideal solenoid* is approached when:
  - the turns are closely spaced
  - the length is much greater than the radius of the turns



Ampere's Law Applied to a Solenoid

- Ampere's law can be used to find the interior magnetic field of the solenoid
- Consider a rectangle with side ℓ parallel to the interior field and side *w* perpendicular to the field
  - This is loop 2 in the diagram
- Only the side of length ℓ inside the solenoid contributes to the integral.
  - This is side 1 in the diagram

Ampere's Law Applied to a Solenoid, cont.

- Applying Ampere's Law gives

$$\oint \vec{B}\cdot d\vec{s} = \int_{path\,1} B\cdot d\vec{s} = B\int_{path\,1} ds = Bl$$

- The total current through the rectangular path equals the current through each turn multiplied by the number of turns

$$Bl = \mu_0 NI \ldots B = \frac{\mu_0 NI}{l}$$

Magnetic Field of a Solenoid, final

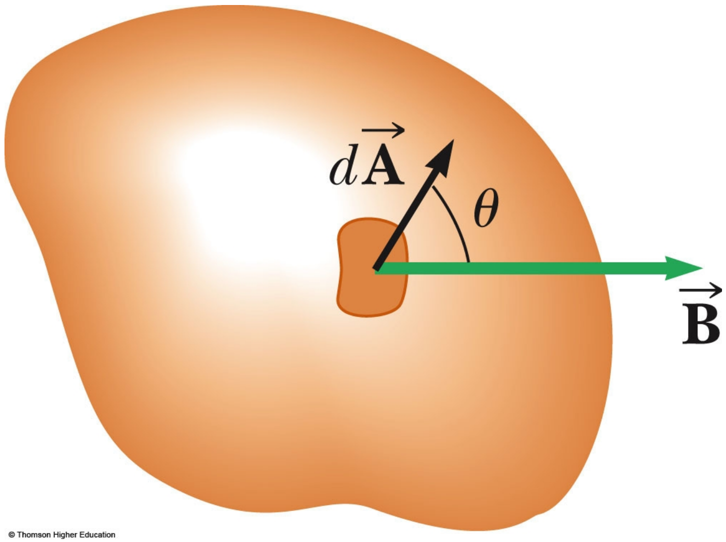
- Solving Ampere's law for the magnetic field is

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

  - $n = N / \ell$ is the number of turns per unit length
- This is most accurate at points near the center of a real solenoid

Magnetic Flux

- The magnetic flux associated with a magnetic field is defined in a way similar to electric flux
- Consider an area element *dA* on an arbitrarily shaped surface



Magnetic Flux, cont.

- The magnetic field in this element is $\vec{B}$
- $d\vec{A}$ is a vector that is perpendicular to the surface
- $d\vec{A}$ has a magnitude equal to the area *dA*
- The magnetic flux Φ_B is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

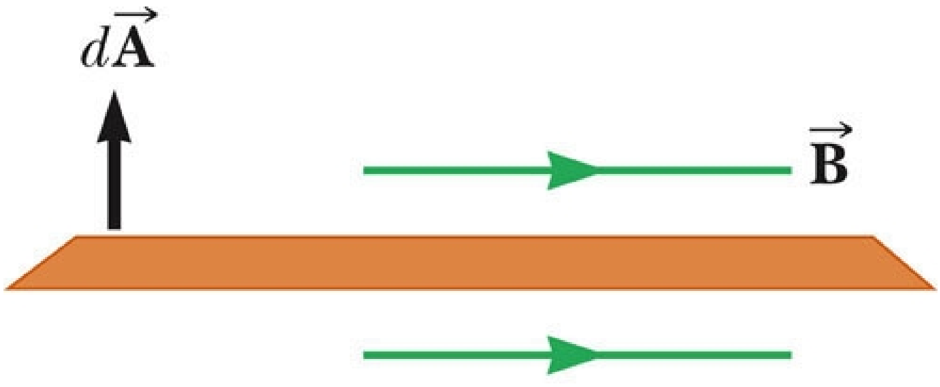
- The unit of magnetic flux is T·m^2 = Wb
  - Wb is a *weber*

Magnetic Flux Through a Plane, 1

- A special case is when a plane of area *A* makes an angle *θ* with $d\vec{A}$

The magnetic flux is $\Phi_B = BA\cos\theta$

- In this case, the field is parallel to the plane and $\Phi_B = 0$



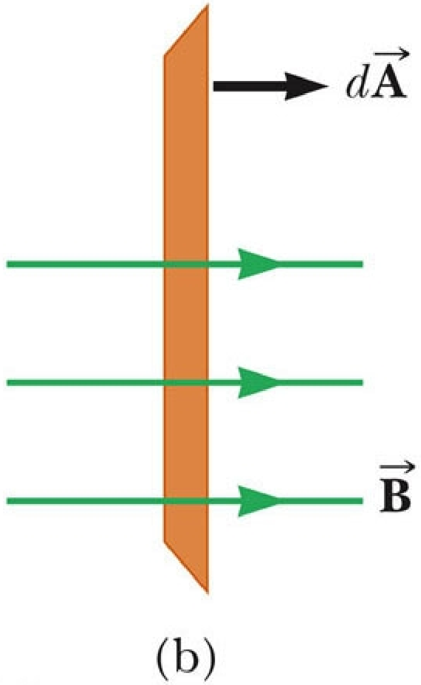
(a)

PLAY ACTIVE FIGURE

Magnetic Flux Through A Plane, 2

- The magnetic flux is $\Phi_B = BA \cos \theta$
- In this case, the field is perpendicular to the plane and

  $$\Phi_B = BA$$
- This will be the maximum value of the flux
- Use the active figure to investigate different angles



(b)

PLAY ACTIVE FIGURE

Gauss' Law in Magnetism

- Magnetic fields do not begin or end at any point
  - The number of lines entering a surface equals the number of lines leaving the surface
- **Gauss' law in magnetism** says the magnetic flux through any closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0$$