Chapter 7

Energy of a System

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Outline for Week 7,D1

Exam I return

Work by a constant force $W = F\Delta r\cos\theta$

Scalar dot products, **A•B** = **C**

Homework

Ch. 7 Read 7.1-7.4

P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55, 56,59,60,65

MiscQ: 1-13 (odd) Do for next Mon

Notes:

"NEW STUFF": added chapter7.pdf, two practice quizzes, and Ch 7 hwk key.

Outline for Week 7,D2

What is energy?

Work by a constant force $W = F \Delta r \cos\theta$

Scalar dot products, **A•B** = C

Work by a variable force $W = \int \vec{F} \cdot d\vec{r}$

Homework

Ch. 7 Read 7.1-7.4

P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55, 56,59,60,65

MiscQ: 1-13 (odd) Do for next Mon (after break)

Notes:

"NEW STUFF": updated chapter7.pdf.

Midterms will be based on Exam 1.

What is Energy?

Some problems are very difficult to solve with Newton's Laws alone, but the *Law of Conservation of Energy* makes them easier.

Energy: in physics, a quantity measured in Joules (SI), ergs, calories, BTU, N-m, ft-lbs, or kW-hrs. It's dimensions are ML^2/T^2. It is a scalar, NOT a vector.

Energy: "the ability to do work" (Energy is not easily defined!)

Types of energy:

Energies stored in a system: thermal (or internal), kinetic (energy of motion), potential (energy of position: gravitational, elastic, nuclear, chemical)

Energies transferred into a system: **work** (W) and heat (Q). (Others: T_{ET} T_{MT} T_{ER})

It is wrong to say "the system contains a lot of work". Instead you can say "work was done on the system". You can say "the system contains a lot of kinetic and potential energy" or "work increased the kinetic energy of the system".

Work

The **work**, *W*, done on a system by a *constant force* is given by:

$$W = F\,\Delta r \cos\,\theta$$

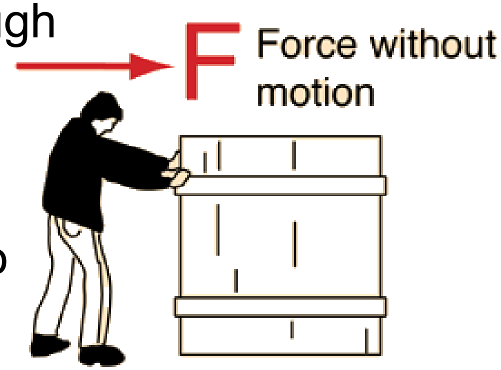
where *F* is the magnitude of the force, Δr is the magnitude of the displacement of the point of application of the force, and θ is the angle between the force and the displacement vectors.

- The meaning of the term *work* is distinctly different in physics than in everyday meaning.
- Work is <u>not</u> a vector, even though it involves two vectors, $\boldsymbol{F}$ and $\Delta\boldsymbol{r}$.
- Work can be + or – because *cos θ* can be + or -.
- Two geometric interpretations:
    - F cos θ is the component of F parallel to Δr, (i.e., $W=F_{\parallel}\Delta r$)
    - Δr cos θ is the component of Δr parallel to F. (i.e., $W=F\Delta r_{\parallel}$)

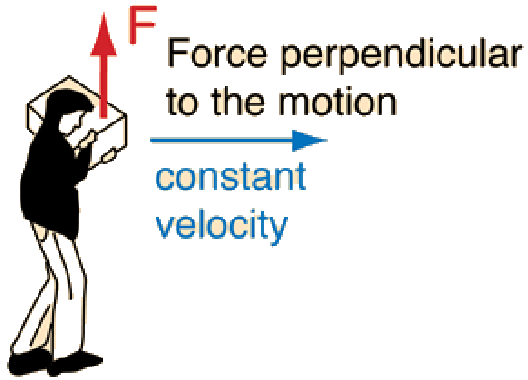
Work, cont.

$W = F\,\Delta r \cos\theta$

- The displacement, $\Delta\mathbf{r}$, is that of the point of application of the force.
- A force does no work on the object if the force does not move through a displacement.
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.



When a force is exerted on an object which does not move, no work is done on the object.



When an object is carried at constant velocity by a force which acts at right angles to the motion, no work is done on the object.

Work, cont.

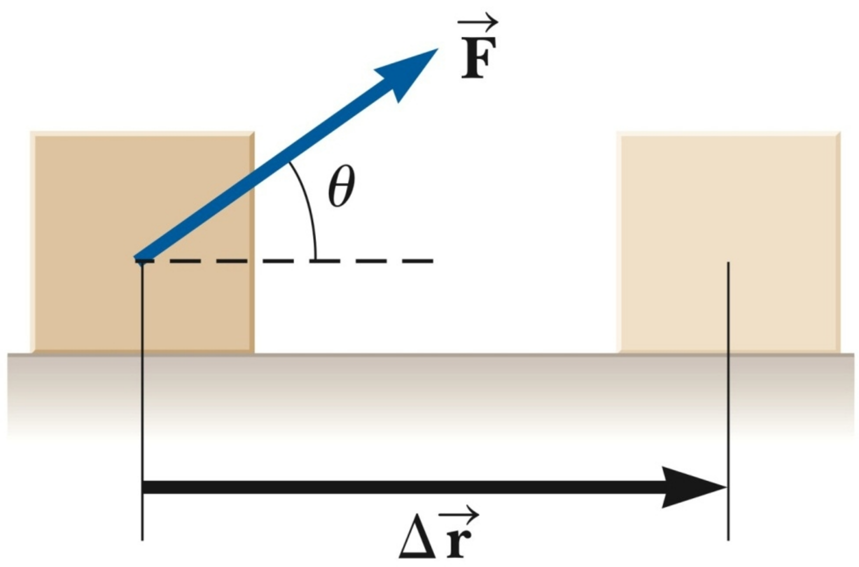
$W = F\,\Delta r \cos\theta$

Q1: What work is done by F =10 N if θ=0 degrees and Δr=2 m?

Q2: What work is done by F=10 N if θ=60 degrees and Δr=2 m?

Q3: What work is done by F=10 N if θ=90 degrees and Δr=2 m?

Q4: The box weighs 20 N. What is the work done by the normal force on the box when θ=0 degrees and Δr=2 m?



Q5: The box weighs 20 N. What is the work done by the force of gravity on the box when θ=0 degrees and Δr=2 m?

Q6: What is the work done by the force of friction on the 20N box if μ_k = 0.5 and when θ=0 degrees and Δr=2 m?

Work, cont.

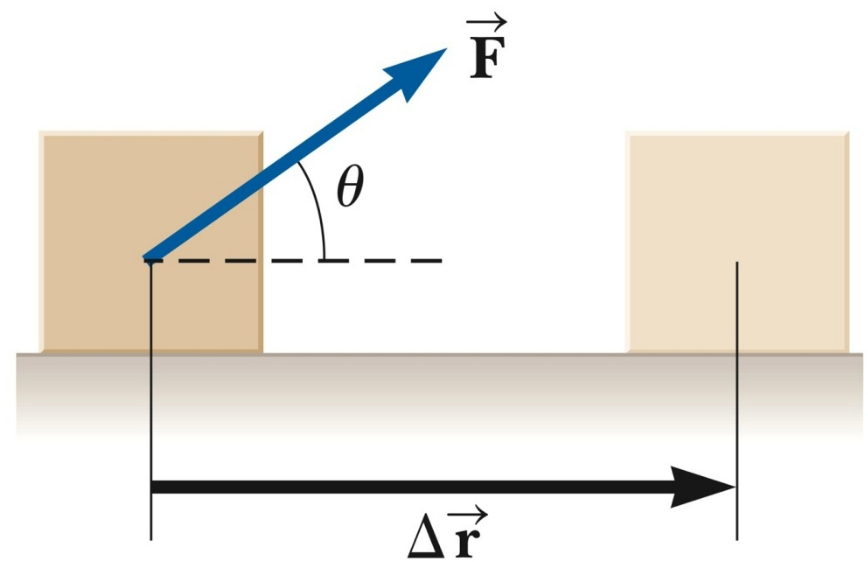
$W = F \, \Delta r \cos \theta$

*Q7: What work is done by **F** =20 N î if Δ**r**=2 m î?*

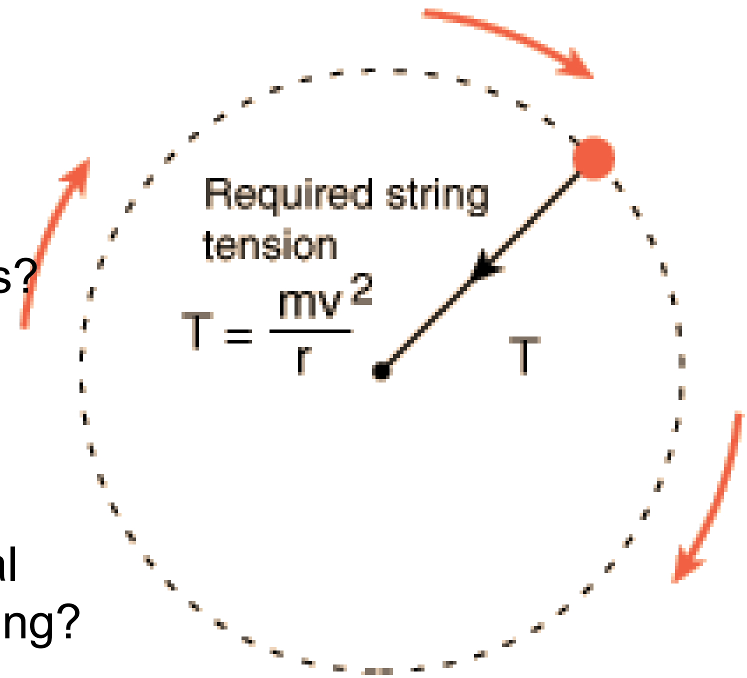
*Q8: What work is done by **F**=-20 N î if Δ**r**=2 m i? How could this happen?*

*Q9: What work is done by **F**=20 î + 15 ĵ N if Δ**r**=3 m î?*

*Q10: What work is done by **F**=20 î + 0 ĵ N if Δ**r**=3 m î – 2m ĵ ?*



Work, cont.



What is the work done by the tension force in 1 revolution of the mass?

What is the work by the tension force in ½ revolution of the mass?

What is the work done by the centripetal force for any angle swept out by the string?

What is the work done by the net force on the mass for any angle swept out by the string?

Displacement in the Work Equation

The displacement is that of the point of application of the force.

If the force is applied to a rigid object that can be modeled as a particle, the displacement is the same as that of the particle.

For a deformable system, the displacement of the object generally is not the same as the displacement associated with the forces applied.

Work by friction is also hard to explain because there is no single point of application. But it can ultimately be modeled by $W = f_k \Delta r \cos \theta$ and is usually negative because θ=180.

Outline for Week 7,D3

Scalar dot products, **A•B** = C

Work by a variable force $W = \int \vec{F} \cdot d\vec{r}$

Homework

Ch. 7 Read 7.1-7.4

P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
56,59,60,65

MiscQ: 1-13 (odd) Do for next Mon (after break)

Notes:

"NEW STUFF": added exam-like questions for Ch.7, updated chapter7.pdf.

Midterms will be based on Exam 1.

Work (cont.)

The normal force and the gravitational force do no work on the object.

- $\cos\theta = \cos 90° = 0$

The force **F** is the only force that does work on the object.

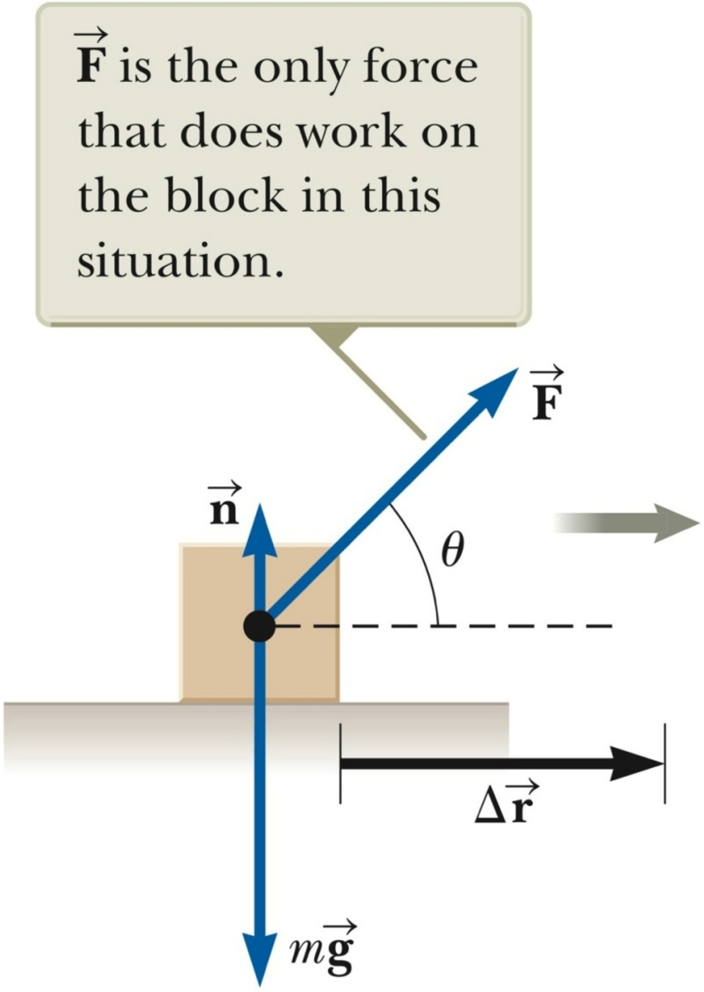
Example with acceleration.

Q: How much work is done pushing a 46 kg crate to the right with $\mathbf{a}$=2.5 m/s$^2\hat{\imath}$ over 10.3 m across a floor with μ_k=0.4?

Sol: First find F_{app}, then $W=F_{app}\Delta r$

$\mathbf{F}_{net}=m\mathbf{a} = \mathbf{F}_{app} + \mathbf{f}_k$

… W = 3042 J.



Work Is An Energy Transfer

If the work done on a system is positive, energy is transferred into the system.

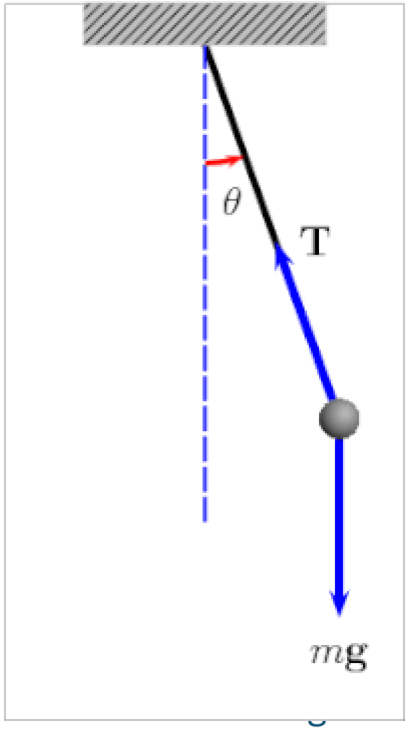
If the work done on the system is negative, energy is transferred out of the system.

Example system: a mass on a string (a pendulum).

Energy that can be stored: gravitational potential energy and kinetic energy

Q: How can you use work to transfer energy in?

Q: How can you use work to transfer energy out?



Scalar (Dot) Product of Two Vectors

The scalar product of two vectors is written as $\vec{A}\cdot\vec{B}$.

- It is also called the dot product.

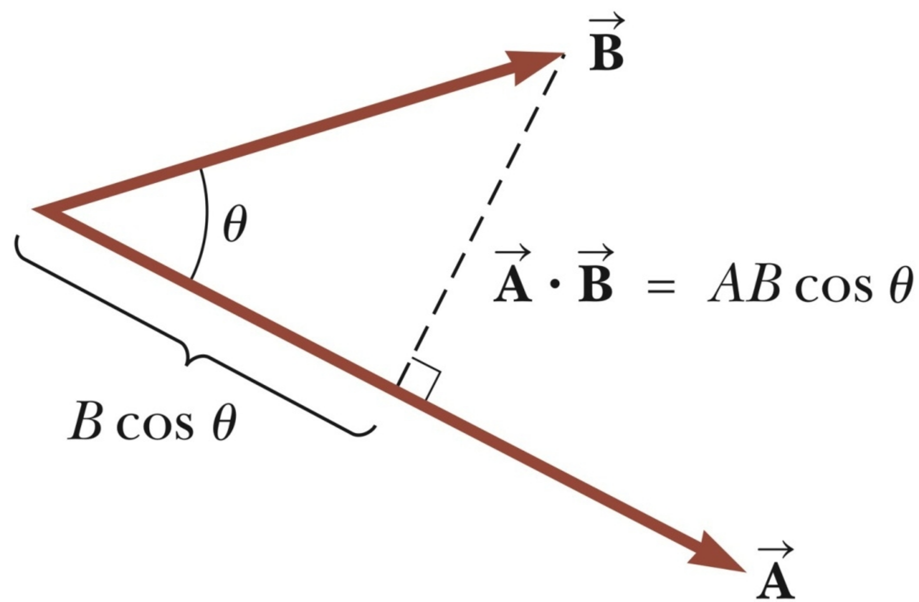
$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

- θ is the angle *between A* and *B*

Applied to work, this means

$$W = F\Delta r\cos\theta = \{\vec{F}\cdot\Delta\vec{r}¿$$

(for the case of constant forces)



Scalar Product, cont

The scalar product is commutative.

- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

The scalar product obeys the distributive law of multiplication.

- $\vec{A} \cdot \left(\vec{B} + \vec{C} \right) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Dot Products of Unit Vectors

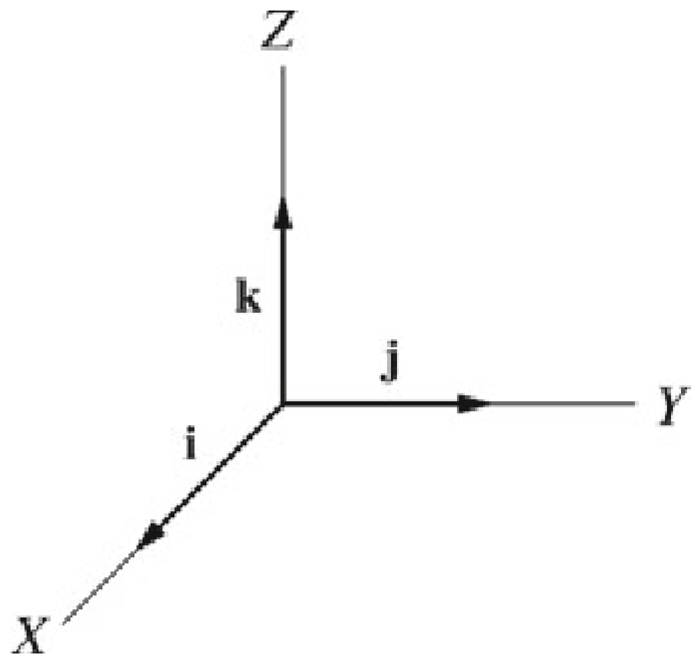
$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$
$$\hat{i}\cdot\hat{j}=\hat{i}\cdot\hat{k}=\hat{j}\cdot\hat{k}=0$$

Using component form with vectors:

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$
$$\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$
$$\vec{A}\cdot\vec{B}=AxBx+AyBy+AzBz$$

In the special case where

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$



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Dot Products - example

$$\vec{A}\cdot\vec{B} = AxBx + AyBy + AzBz$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Ex) (P. 20) Find the angle between **A**=5.8i-3.5j-6.2k
and **B**=8.2i+4.3j-7.0k

$$\vec{A}\cdot\vec{B} = 5.8\cdot 8.2 + (-3.5)\cdot 4.3 + (-6.2)\cdot(-7.0)$$

A·B = 75.91

Find |**A**| = 9.145 and |**B**|=11.61

Then |**A**||**B**| cos θ = 75.91

cos θ = 75.91/(9.145*11.61) → θ = 44.4 ° (fixed)

Work Done by a Varying Force

$$W = \int \vec{F} \cdot d\vec{r}$$

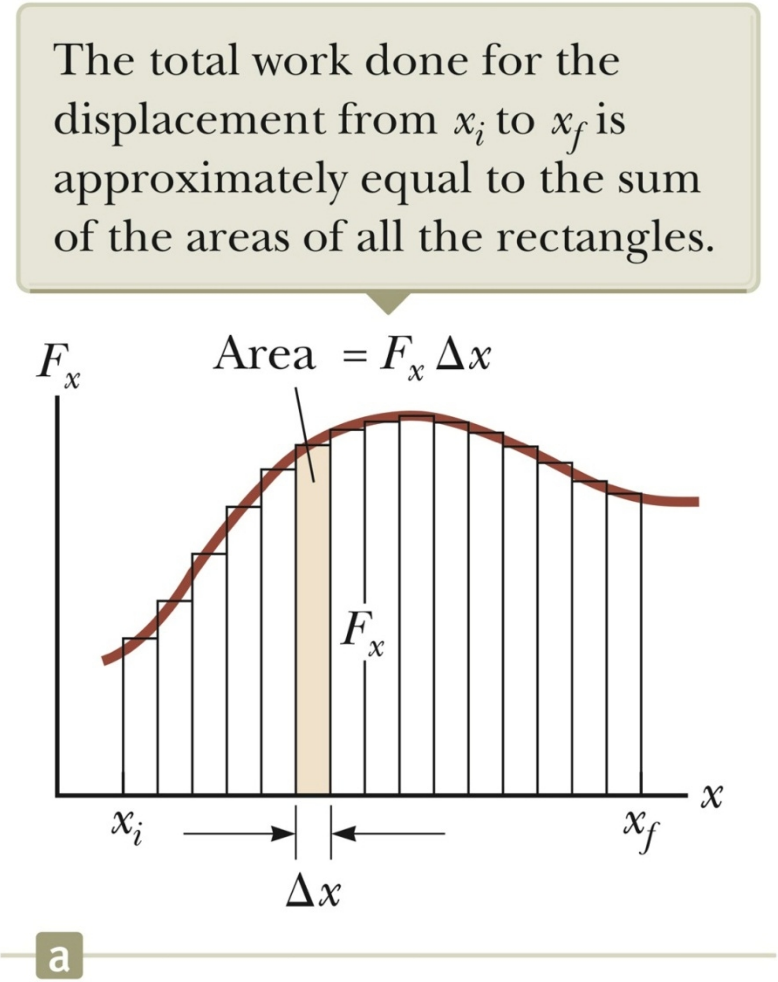
To use W = F Δ r cos θ, the force must be constant, so the equation cannot be used to calculate the work done by a varying force.

Assume that during a very small displacement, Δ*x*, *F* is constant.

For that displacement, $W \sim F\,\Delta x$

For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont.

Let the size of the small displacements approach zero .

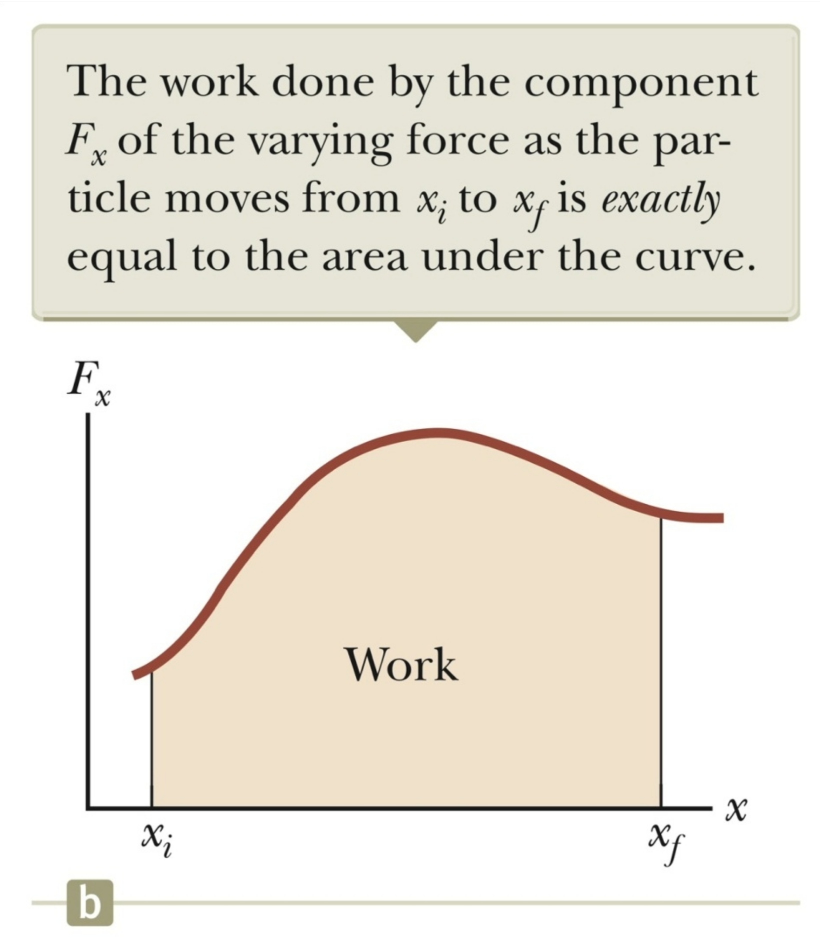
Since

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Or, more generally,

$$W = \int \vec{F} \cdot d\vec{r}$$

The work done is equal to the area under the curve between x_i and x_f.



Work Done by a Varying Force, cont.

$$W = \int \vec{F} \cdot d\vec{r}$$

Example) P. 7.41. Find the work done on a particle moving according to this graph …

a) between x=0 to x=10 m
Ans: 2800 J

b) between x=0 to x=15 m
Ans: 2100 J

Work Done by a Varying Force, cont.

$$W = \int \vec{F} \cdot d\vec{r}$$

Example) P. 7.45. $F(x) = \dfrac{A}{\sqrt{x}}$

With A = 3.00 $\mathrm{Nm}^{1/2}$ what is the work done by this force when the object moves from $x_i=0$ to $x_f=1.0$?

Work Done By Multiple Forces

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum \mathbf{F} \right) d\vec{\mathbf{r}}$$

The subscript "ext" indicates the work is done by an *external* agent on the system.

Work Done by Multiple Forces, cont.

If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{\text{ext}} = \sum_{forces} \left(\mathbf{F} \cdot d\mathbf{r} \right)$$

- Remember work is a scalar, so this is the sum of a bunch of scalars.

Outline for Week 8,D1

Work by the spring force $W = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$

The work-KE principle $W_{ext} = \Delta K$

Homework

Ch. 7 Read 7.1-7.4

P. 1,2,5,8,12,18,20,22,25,37,39,41,45,55,
56,59,60,65 MiscQ: 1-13 (odd) Do for today

Ch. 8 Read 8.1-8.6,8.8-8.9 Do P. 1, 3, 4, 6, 9,12,13,17,19,28,
29,30,55,56,58,61,73 for next Monday

Notes: Lab on “Cons. Of Energy”

“NEW STUFF”: updating chapter7.pdf.

Midterms are based on Exam 1.

Last time: Work Done by a Varying Force

Can't use $W = F\Delta r\cos\theta = \{\vec{F}\cdot\Delta\vec{r}$ ¿

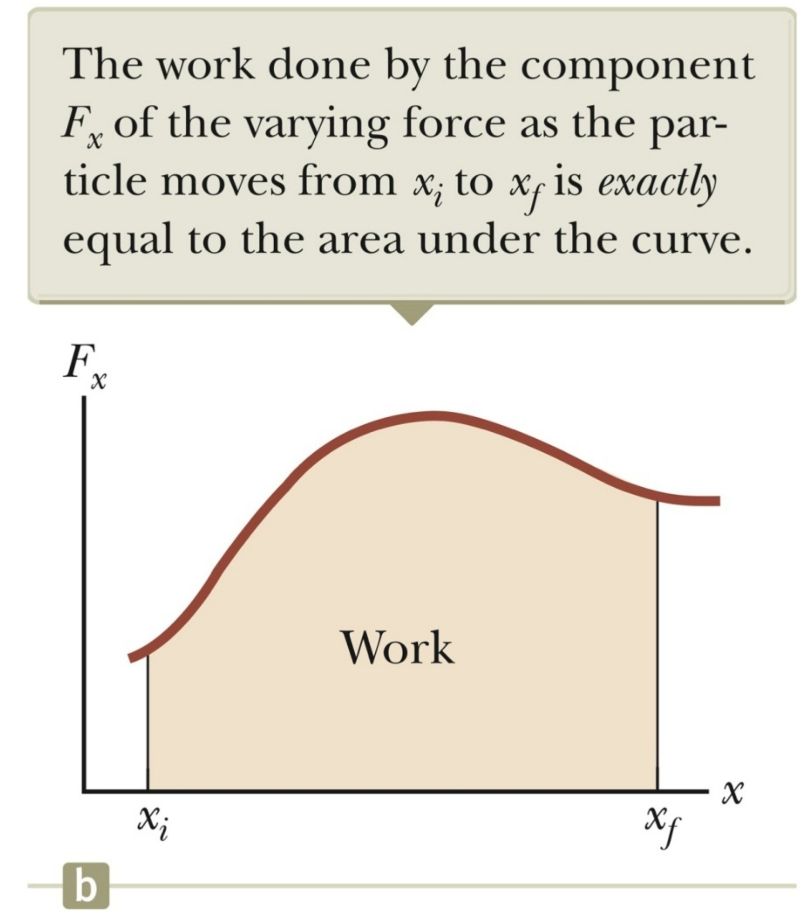
But you can add up a bunch of ***F***·Δ***r*** terms to approximate the work. Or, do an integral for the exact right amount:

$$\lim_{\Delta x\to 0}\sum_{x_i}^{x_f} F_x\Delta x = \int_{x_i}^{x_f} F_x dx$$

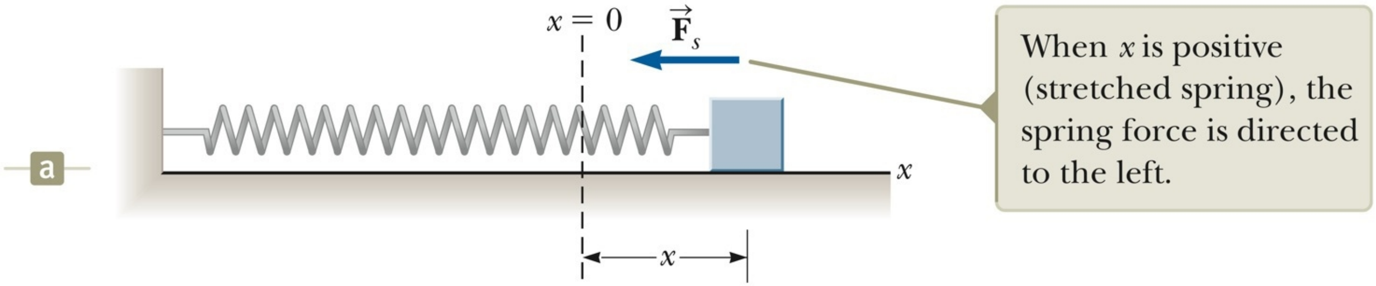
Or, more generally,

$$W = \int \vec{F}\cdot d\vec{r}$$

The work done is equal to the area under the curve between x_i and x_f.



The Spring Force is a variable force!



In order to calculate the work done by the spring force, we need to know how the force varies with position, which is given by ...

Hooke's Law:

$$F_s = -kx$$

F_s is the spring force,
k is the spring constant (N/m), and
x is the distance of [the block] from the equilibrium position.

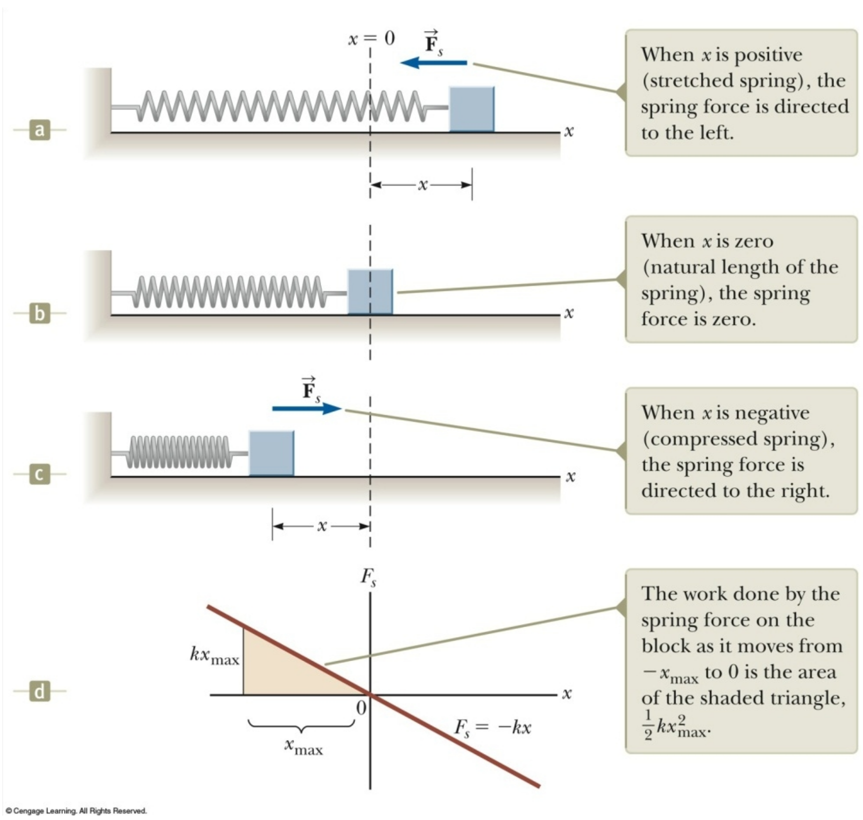
The "-" indicates that the force is in the opposition direction of the displacement from equilibrium.

The block is usually on a horizontal, frictionless surface.

Work Done By A Spring

Observe the motion of the block with various values of the spring constant.

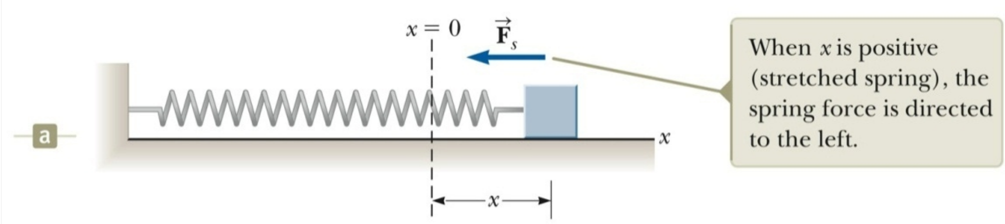
See 7.9.swf



Hooke's Law, cont.

The vector form of Hooke's Law is

$$\mathbf{F}_s = F_x \hat{\mathbf{i}} = -kx\hat{\mathbf{i}}$$



When x is positive (spring is stretched), F is negative

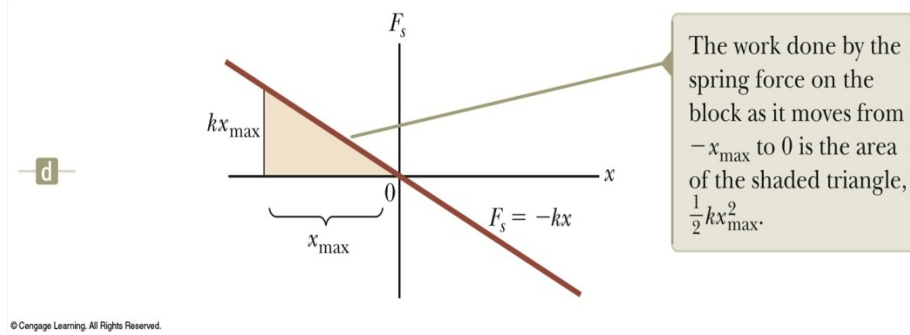
When x is 0 (at the equilibrium position), F is 0

When x is negative (spring is compressed), F is positive

Work Done by a Spring

Identify the block as the system.

Ex) Calculate the work by the spring as the block moves from x_i = - x_{max} to x_f = 0.



$$W_s = \int \vec{F}_s \cdot d\vec{r} = \int_{x_i}^{x_f} \left(-kx\,\hat{i}\right)\cdot\left(dx\,\hat{i}\right)$$

$$W_s = \int_{-x_{max}}^{0} -kx\,dx = \frac{1}{2}(-k)\left(0^2 - (-x_{max})^2\right) = \frac{1}{2}kx^2_{max}$$

Notice that this is the area of the shaded triangle!

Q: What is the work done by the spring if the block moves from 0 to x_{max}?

Q: What is the work done by the spring if the block moves from $-x_{max}$ to x_{max} ?

Q: What is the work done by the spring if the block moves from

X_i=-0.5 to x_f=0.3 m if k=300 N/m?

Work Done by a Spring, cont.

Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_{f.}$

The work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- Or $W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$
- If the motion ends where it begins, W = 0

Spring with an Applied Force

Suppose an external agent, F_{app}, like someone's hand, stretches the spring.

The applied force is equal and opposite to the spring force (if block's speed constant).

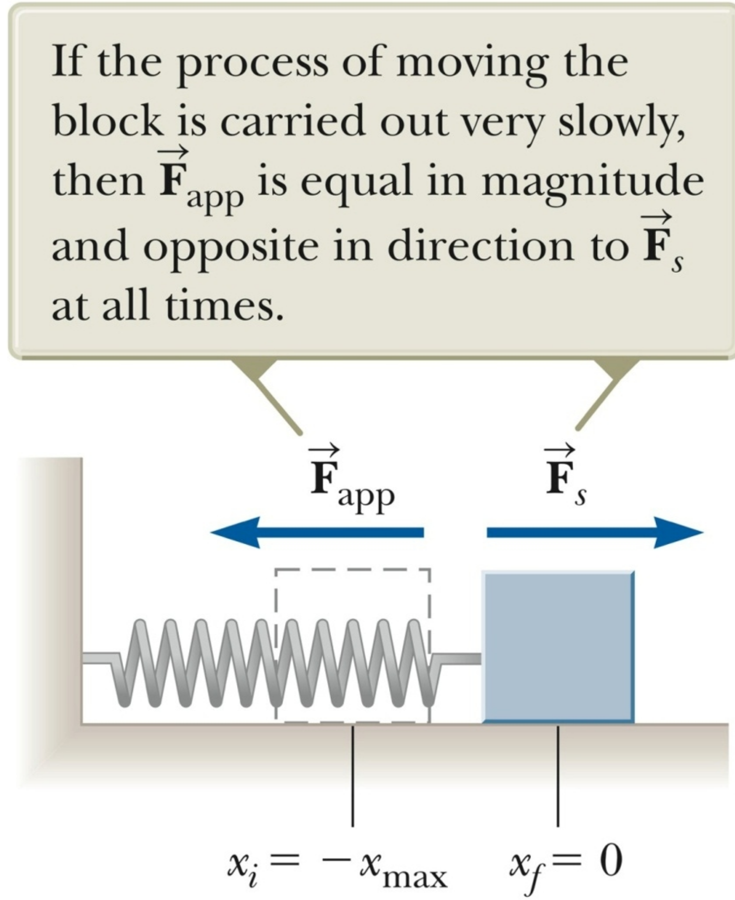
$$\mathbf{F}_{app} = F_{app}\hat{\mathbf{i}} = -\mathbf{F}_s = -\left(-kx\hat{\mathbf{i}}\right) = kx\hat{\mathbf{i}}$$

Work done by F_{app} as the block moves from $-x_{max}$ to $x = 0$ is equal to

$-\frac{1}{2} kx^2_{max}$

For any displacement, the work done by the applied force (at const speed) is

$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$



Spring with an Applied Force

For any displacement, the work done by the applied force is

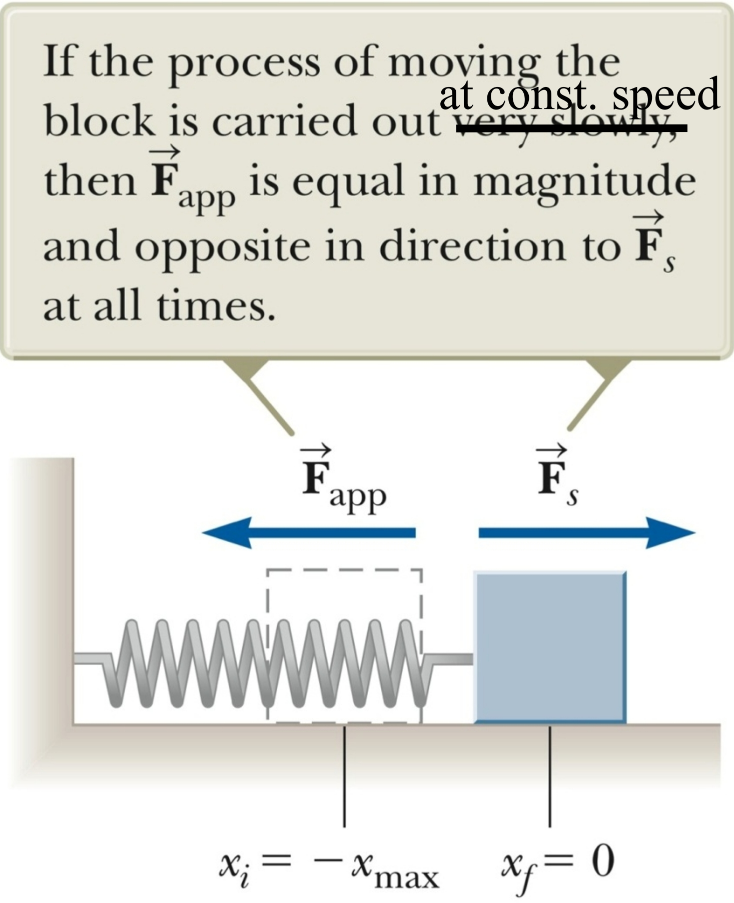
$$W_{app} = \int_{x_i}^{x_f} (kx)\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Q: What is the work done by a hand pushing the block at <u>constant speed</u> from x_i=0 to x_f=0.5 m if k=300 N/m?

Ans: ½(300)(0.5)^2 = 37.5 J

Q: What is the net work done by a hand pushing the spring-block at constant speed from x_i=-0.5 m to x_f=0.5 m if k=300 N/m?

Ans: ½(300)(0.5^2-0.5^2) = 0 J



Spring with an Applied Force

For any displacement, the work done by the applied force is

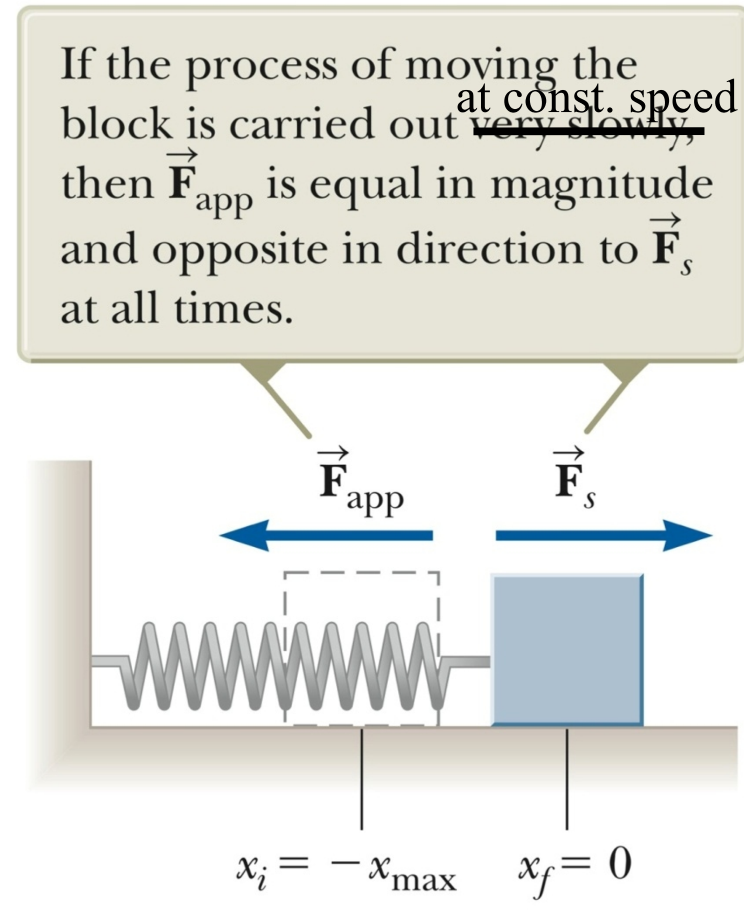
$$W_{app} = \int_{x_i}^{x_f} (kx)\, dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Q: What is the work done by a hand pushing the block at <u>constant speed</u> from $x_i=0$ to $x_f=0.5$ m if k=300 N/m?

Ans: $\frac{1}{2}(300)(0.5)^2 = 37.5$ J

Q: What is the net work done by a hand pushing the spring-block at constant speed from $x_i=-0.5$ m to $x_f=0.5$ m if k=300 N/m?

Ans: $\frac{1}{2}(300)(0.5^2-0.5^2) = 0$ J



Outline for Week 8,D2

Finish work-KE principle $W_{ext} = \Delta K$

Conservative and non-conservative forces

Potential energy (gravity, elastic, general)

Homework

Ch. 8 Read 8.1-8.6,8.8-8.9 Do P. 1, 3, 4, 6, 9,12,13,17,19,28, 29,30,55,56,58,61,73 for next Monday

Notes:

Lab on “Cons. Of Energy”

“NEW STUFF”: Added Practice Quiz on Ch. 8.

Updated chapter7.pdf again.

Kinetic Energy

One possible result of work acting as an influence on a system is that the system changes its speed.

The system could gain or lose *kinetic energy.*

Kinetic Energy is the energy of an object due to its motion.

- $K = \frac{1}{2} mv^2$
  - K is the kinetic energy
  - m is the mass of the particle
  - v is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into (or out of) a system.

Work - Kinetic Energy principle

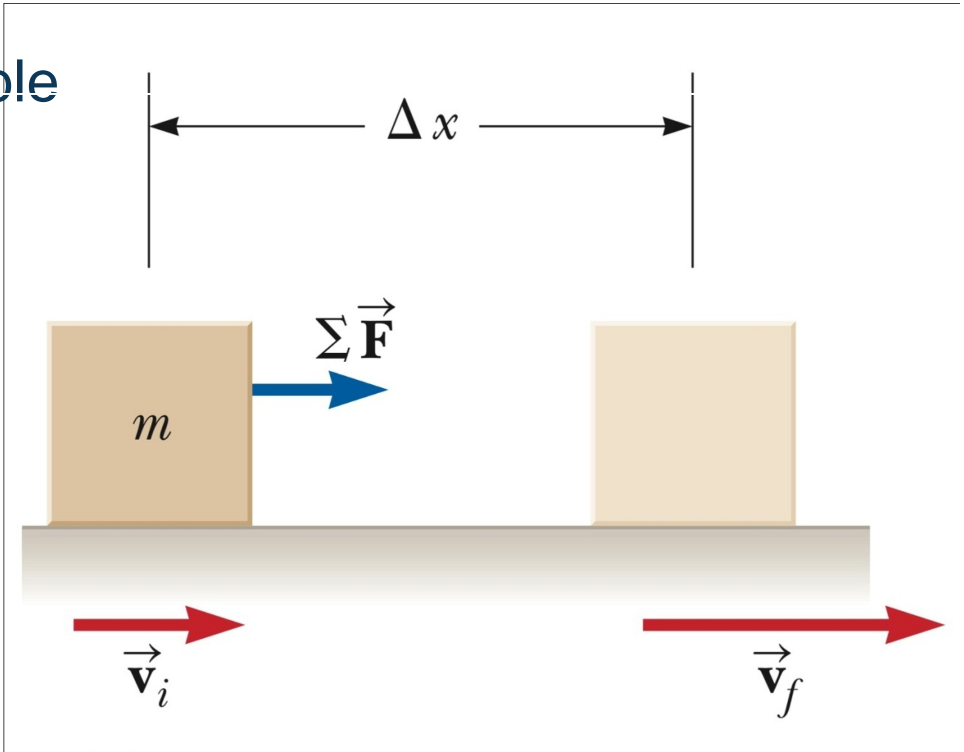
Derivation:

$$W_{ext} = \int_{x_i}^{x_f} \sum F\,dx = \int_{x_i}^{x_f} ma\,dx$$

$$W_{ext} = \int_{v_i}^{v_f} mv\,dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{ext} = K_f - K_i = \Delta K$$



Ex) Find speed of a frictionless block (m=0.5 kg) at x=0 if it is attached to a spring with k=300 N/m and released from rest at x=0.3 m.

Ans: $W_{ext}=W_s=-½k(0-0.3^2)=½\ m\ (v_f^2-0)$ … v_f = 7.35 m/s

Ex) (P. 60) An 85 g arrow is fired from a bow whose string exerts an average force of 105 N over a distance of 75 cm. What is the speed of the arrow as it leaves the bow?

Ans: $W_{ext}=1/2\ m\ (v_f^2-0)$ with $W_{ext}=F_{avg}\Delta x=105(.75m)$. v_f = 43 m/s

Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states $W_{ext} = K_f - K_i = \Delta K$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

- The speed of the system increases if the work done on it is positive.
- The speed of the system decreases if the net work is negative.
- Also valid for changes in rotational speed

The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.

Ex) You exert a 20 N upward force on an object that weighs 10 N. The force is applied over a distance of 0.3 m and the object starts at rest. What is the change in KE of the object? Ans: the work is also changing the gravitational potential energy of the object, so $W_{hand} \neq \Delta KE$. Instead, $W_{hand} = \Delta KE + \Delta PE$, or $W_{hand+grav} = \Delta KE$. Using the latter: **20N(0.3)-10N(0.3)=ΔKE=3J**.

Work-Kinetic Energy Theorem – Example

Find the final velocity of the block.

The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.

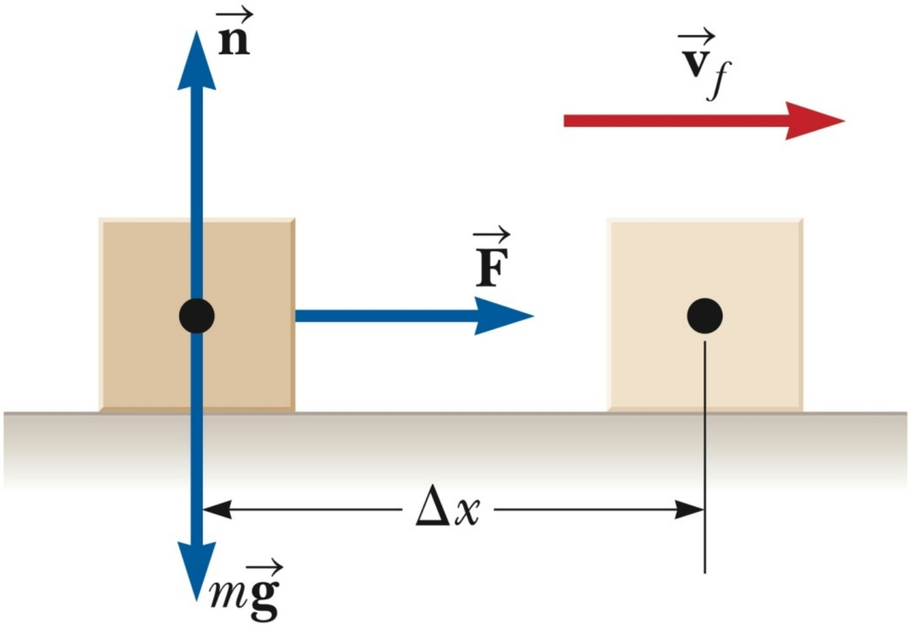
$W_{ext} = \Delta K = ½\ mv_f^2 – 0$

$F\Delta x = ½mv_f^2 \rightarrow$ $v_f = (2F\Delta x/m)^{½}$

The answer for v_f could be checked using the kinematic equations …

Substitute $a = F_{net}/m$ into

$v_f^2 - v_i^2 = 2a(\Delta x)$, again giving $v_f = (2F\Delta x/m)^{½}$



What if friction, f_k, acted in the opposite direction of the applied force?

Ans: you modify the Work-KE Theorem to be

$W_{ext} = \Delta K + f_k\Delta x$. This gives $v_f = (2[F-f_k]\Delta x/m)^{½}$

Conservative Forces

The work done by a conservative force on a particle moving between any two points is *independent of the path* taken by the particle.

The work done by a conservative force on a particle moving through any *closed* path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity (DEMO: 7.15.swf)
- Spring force

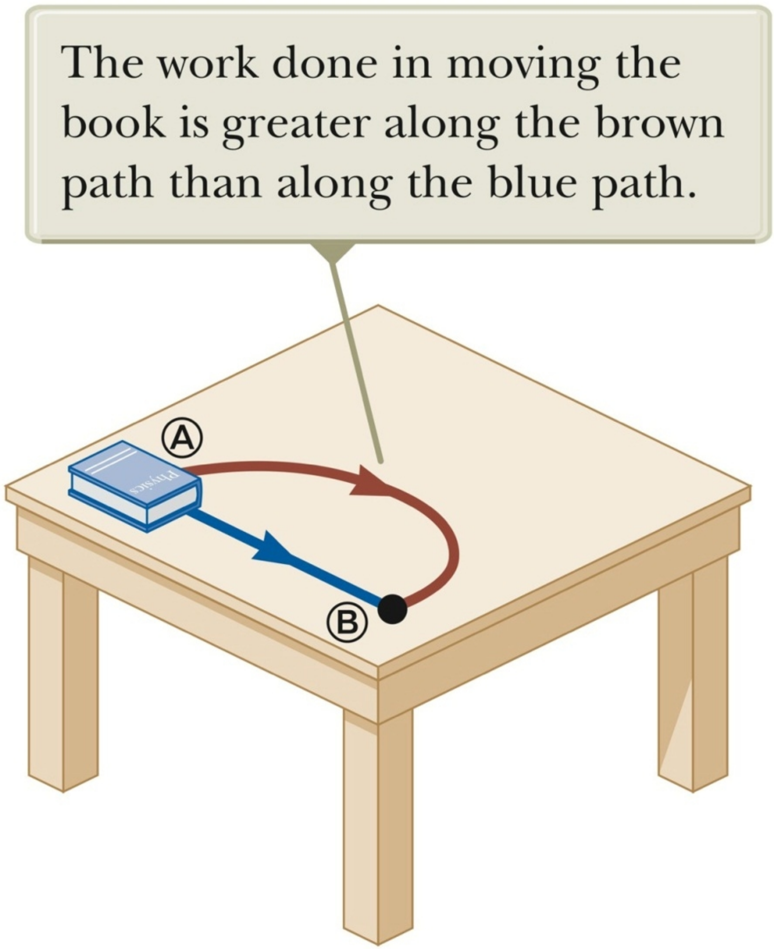
Examples of non-conservative forces:

- Friction
- Air resistance
- An applied force which counters friction.

Non-conservative Forces, cont.

The work done against friction is greater along the brown path than along the blue path.

Because the work done depends on the path, friction is a non-conservative force.



Outline for Week 8,D3

Conservative and non-conservative forces (cont.)
Potential energy (gravity, elastic, general)

Homework

Ch. 8 Read 8.1-8.6,8.8-8.9 Do P. 1, 3, 4, 6, 9,12,13,17,19,28, 29,30,55,56,58,61,73 for next Monday

Notes:

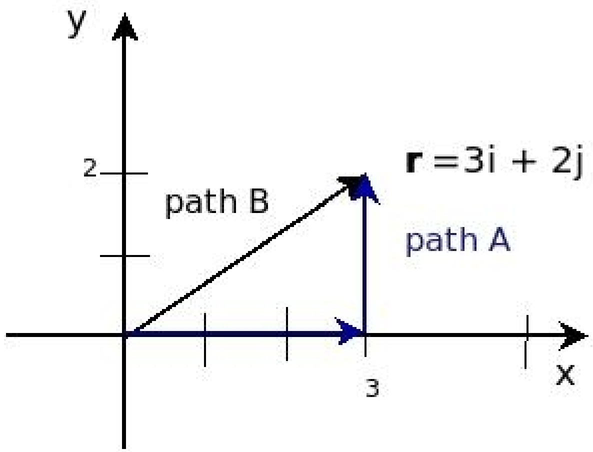
Lab on "Cons. Of Energy"
"NEW STUFF": Added Practice Quiz on Ch. 8.
Updated chapter7.pdf again.

Conservative Forces (cont.)

Ex) Show that the work done by gravity on mass m moved from (0,0) to (3,2) is the same for 2 straight-line paths.

Path A: (0,0) to (3,0) to (3,2)

Path B: (0,0) to (3,2)



$$W_A = \int_{(0,0)}^{(3,0)} \vec{F}_g \cdot d\vec{r} + \int_{(3,0)}^{(3,2)} \vec{F}_g \cdot d\vec{r}$$

$$W_A = \int_{(0,0)}^{(3,0)} -mg\,\hat{j} \cdot dx\,\hat{i} + \int_{(3,0)}^{(3,2)} -mg\,\hat{j} \cdot dy\,\hat{j} \quad \longrightarrow \quad W_A = 0 - 2mg$$

$$W_B = \int_{(0,0)}^{(3,2)} \vec{F}_g \cdot d\vec{r} = \vec{F}_g \cdot \Delta\vec{r}$$

$$W_B = mg(-\hat{j}) \cdot (3\hat{i} + 2\hat{j}) \quad \longrightarrow \quad W_B = -2mg$$

Non-conservative Forces

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the **mechanical energy** of the system.

$E_{mech} = K + U$

- K includes the kinetic energy of all moving members of the system.
- U includes all types of potential energy in the system.

Conservation of mechanical energy:

$\Delta E_{mech} = \Delta K + \Delta U = 0$ for a closed system with no non-conservative forces

$\Delta E_{mech} = \Delta K + \Delta U < 0$ for a closed system with non-conservative forces

$\Delta E_{mech} = -f_k d$ for a closed system with non-conservative forces

Conservative Forces, cont

We can associate a potential energy for a system with any conservative force acting between members of the system.

- This can be done only for conservative forces.
- In general: $W_{int} = -\Delta U$
  - W_{int} is used as a reminder that the work is done by one member of the system on another member and is internal to the system.
- Work done on a component of a system by a conservative force internal to an isolated system causes a decrease in the potential energy of the system.

Example: make energy equations for a falling apple when the system is the Earth+apple, and when the system is just the apple.

Conservative Forces and Potential Energy

Define a potential energy function, *U*, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, *F*, is

$$W_{\text{int}} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

- ΔU is negative when *F* and *x* are in the same direction (e.g., a falling rock)

Ex) Find the potential energy function for a spring.

Ans: $U_s = \frac{1}{2}kx^2$

Ex) Find the potential energy function for gravity.

Ans: $U_g = mgy$

Conservative Forces and Potential Energy

The conservative force can be found from the potential energy function through

$$F_x = -\frac{dU}{dx}$$

The *x* component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to *x.*

- Can be extended to three dimensions:

$$F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

Ex) (P.8) Find the force associated with $U(x,y,z)=3x^2+2xy+4y^2z$.

Ans: use the above derivatives to find

$F_x = -(6x+2y)$, $F_y=-(2x+8y)$, $F_z=-4y^2$

Then the force is $\vec{F}(x,y,z)=(-6x-2y)\hat{i}+(8y-2x)\hat{j}-4y^2\hat{k}$

Conservative Forces and Potential Energy – Check

Ex) Find the force of a stretched spring from its potential energy, U_s:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for $U_s = \frac{1}{2}kx^2$

U is an important function because the conservative force can be derived from it.

Gravitational Potential Energy

The system is the Earth and the book.

$\mathbf{F}_{app}$ is external, $\mathbf{F}_g$ is internal.

Suppose $\mathbf{F}_{app}$ does positive work on the book, lifting it at <u>constant speed</u> through a vertical displacement.

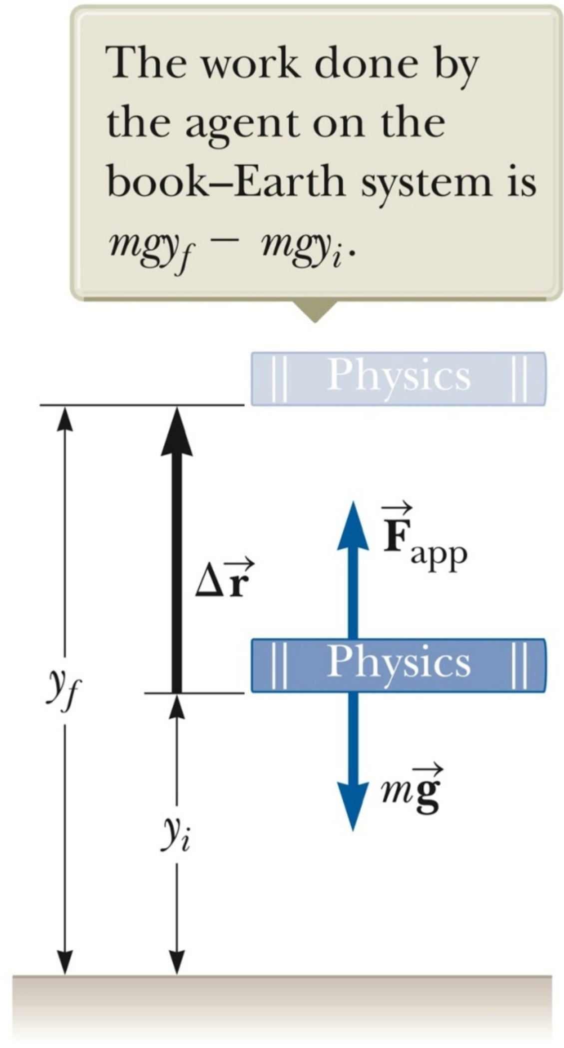
$$\Delta\mathbf{r} = (y_f - y_i)\,\hat{\mathbf{j}}$$

Then $W_{ext} = \mathbf{F}_{app} \cdot \Delta\mathbf{r} = mg\Delta r$

The positive work done on the system by the hand appears as an increase in the grav potential energy of the system.

$$\Delta U_g = W_{ext}$$

Negative work by the hand results in a decrease in the system's energy.



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Gravitational Potential Energy, cont

Assume the book in fig. 7.15 is allowed to fall at a constant speed.

There is no change in kinetic energy since the book starts and ends at the same speed.

Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth.

$$W_{ext} = \left(\mathbf{F}_{app}\right) \cdot \Delta\mathbf{r}$$

$$W_{ext} = (mg\hat{\mathbf{j}}) \cdot \left[\left(y_f - y_i\right)\hat{\mathbf{j}}\right]$$

$$W_{ext} = mgy_f - mgy_i$$

Conservation of mechanical energy with gravity but no friction is:

$\Delta E_m = \Delta K + \Delta U_g = 0$ where $\Delta U_g = W_{ext}$ is work by F_{app}

Ex) (P. 13)

Gravitational Potential Energy, final

The quantity *mgy* is identified as the gravitational potential energy, $U_{g.}$

- $U_g = mgy$

U_g is a scalar in units of Joules (J).

$U_g = mgy$ only applies close to the Earth's surface.
y increases away from the Earth, and y=0 at a convenient reference point, like the Earth's surface.

Work may change the gravitational potential energy of the system.

$W_\mathrm{ext} = \Delta U_\mathrm{g}$ and $W_{int} = -\Delta U_\mathrm{g}$

Potential energy is always associated with a system of two or more interacting objects.

NOTE: we're skipping section 8.7, which gives the expression for gravitational potential energy far from the surface of the Earth: $U(r)=-GM_Em/r$

Outline for Week 9,D1

Example problems in conservation of energy

Stable and unstable equilibrium in U(x) vs x plots.

Power

Homework

Ch. 8 Read 8.1-8.6,8.8-8.9 Do P. 1, 3, 4, 6, 9,12,13,17,19,28, 29,30,55,56,58,61,73 for today

Ch. 9 P. 1,2,4-6,8,9,18,21,23,28,29,37,38,47,54,55 MisConQ. 1-13 (odd) for next Monday

Notes:

Lab: “Cons. Of Linear Momentum” $\mathbf{p} = m\mathbf{v}$

Will make exam-like questions for Ch. 8.

Elastic Potential Energy [Redundant]

Elastic Potential Energy is associated with a spring.

The force the spring exerts (on a block, for example) is $F_s = -kx$

The work done by an external applied force on a spring-block system is

- $W = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_i^2$
- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.
- That expression is the *elastic potential energy*, $\mathbf{U_s = \frac{1}{2} kx^2}$

Elastic Potential Energy, cont.

[Redundant]

This expression is the elastic potential energy:

$$U_s = ½\, kx^2$$

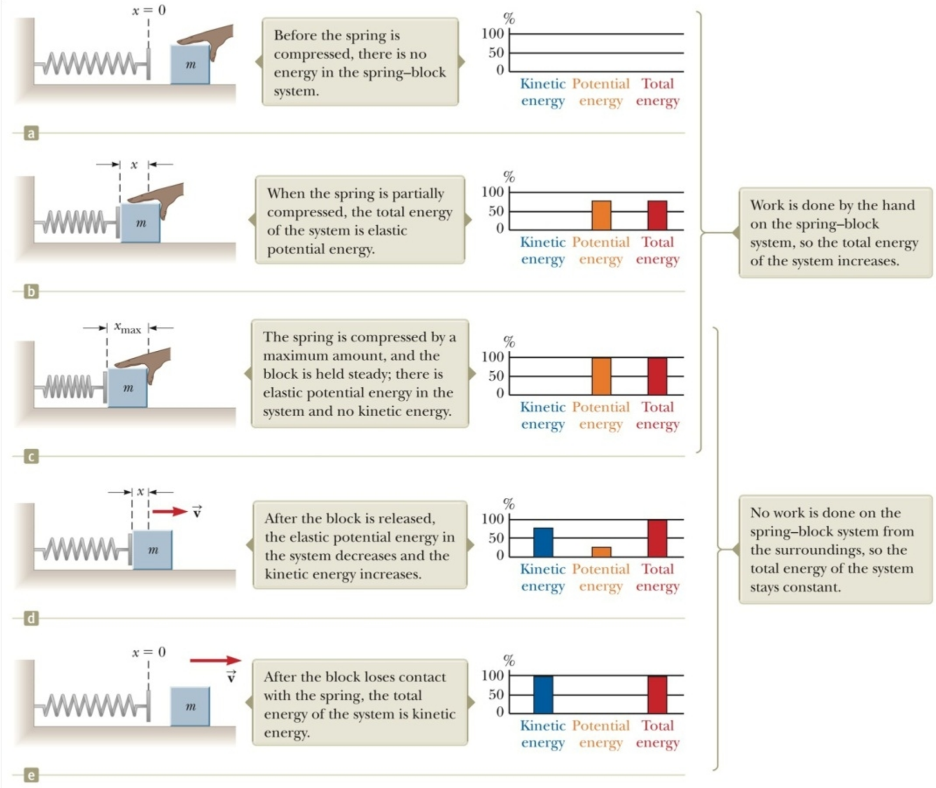
The elastic potential energy can be thought of as the energy stored in the deformed spring, or rubber band.

The stored potential energy can be converted into kinetic energy.

$\Delta E_m = \Delta K + \Delta U_s = 0$ (if no friction), so

$$\Delta K = -\Delta U_s$$

Observe the effects of different amounts of compression of the spring in **7.20.swf**



Elastic Potential Energy, final [Redundant]

The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$).

- The energy is stored in the spring only when the spring is stretched or compressed.

The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.

The elastic potential energy is always positive.

- x^2 will always be positive.

Example problems in conservation of energy.

$\Delta E_{mech} = \Delta K + \Delta U = 0$ for a closed system with no non-conservative forces

Useful for frictionless incline, roller coaster and ski ramp problems.

Ex) (P. 13) A sled is initially given a shove up a frictionless 18.0° incline. It reaches a maximum vertical height 1.22 m higher than where it started at the bottom. What was its initial speed?

Ex) (P. 19) A vertical spring (ignore its mass), whose spring constant is 875 N/m, is attached to a table and is compressed down by 0.220 m. (a) What [maximum] upward speed can It give to a 0.380 kg ball when released? (b) How high above its original position (spring compressed) will the ball fly?

Conservative Forces, final (advanced)

You now know that the spring force, Fs=-kx, and the force of gravity, F_g=mg, are conservative forces, and their corresponding potential energies are:

U_s = ½ kx^2 and U_g=mgy.

Is there a way to tell if a general force, F(x,y,z) is conservative? Ans: yes!

If **F**(x,y,z)= F_x(x,y,z)î + F_y(x,y,z)ĵ + F_z(x,y,z)k, it is conservative if:

$$\frac{\partial F_x}{\partial y}=\frac{\partial F_y}{\partial x} \text{ and } \frac{\partial F_z}{\partial x}=\frac{\partial F_z}{\partial y}=0\text{; or } \frac{\partial F_x}{\partial z}=\frac{\partial F_z}{\partial x} \text{ and } \frac{\partial F_y}{\partial x}=\frac{\partial F_y}{\partial z}=0$$

Ex) $\mathbf{F}_s$=-kxi + 0j + 0 k has all partial derivatives = 0, so it is conservative.

Ex) $\mathbf{F}_g$= 0i -mg j +0 k has all partial derivatives = 0, so it is conservative.

Ex) **F**(x,y) = yi + xj has $\frac{\partial F_x}{\partial y}=\frac{\partial F_y}{\partial x}=1$, so it is conservative.

Ex) **F**(x,y) = xyi + xj has $\frac{\partial F_x}{\partial y}=x$ and $\frac{\partial F_y}{\partial x}=1$, so it is NOT conservative.

Internal Energy and friction

The energy associated with an object's temperature is called its *internal energy*, $E_{int.}$

Let the book and the surfaces be the system. Then, the friction does work and increases the internal energy of the surfaces. $\Delta E_m + \Delta E_{int} = 0$ or $\boldsymbol{\Delta E_m = -f_k d}$

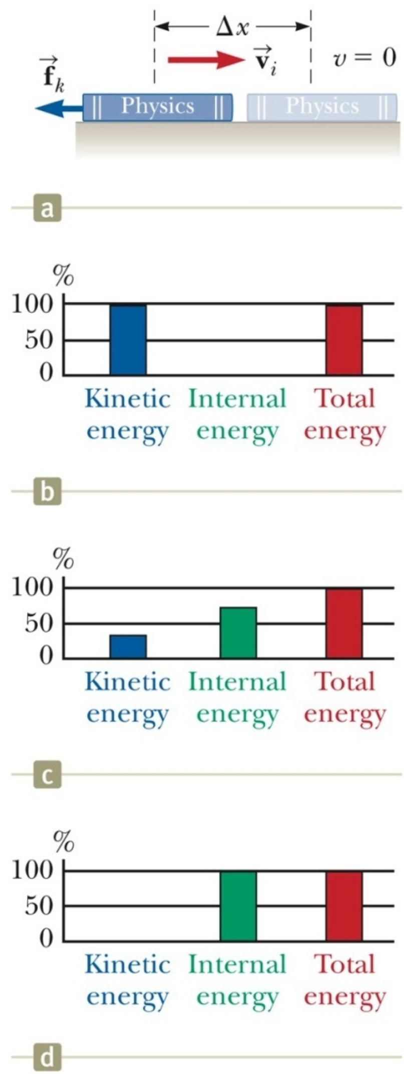
When the book stops, all of its kinetic energy has been transformed to internal energy.

The total energy remains the same.

See **7.18.swf**

If a hand pushes it, $W_{ext} \neq 0$ and

$\Delta E_m + \Delta E_{int} = W_{ext}$ or $\boxed{\boldsymbol{\Delta E_m = W_{ext} - f_k d}}$



Including friction in energy problems

A good general equation for including both friction and other external forces is:

$\boxed{\Delta \mathbf{E}_m = \mathbf{W}_{ext} - \mathbf{f}_k \mathbf{d}}$ where $\Delta \mathbf{E}_m = \Delta \mathbf{K} + \Delta \mathbf{U}$

P. 28) A 16 kg child descends a slide 2.20 m high and starting from rest, reaches the bottom with a speed of 1.15 m/s. How much thermal energy due to friction was generated in this process?

Soln: $\Delta K + \Delta U_g = -f_k d$ ($f_k d$ is the "thermal energy due to friction")

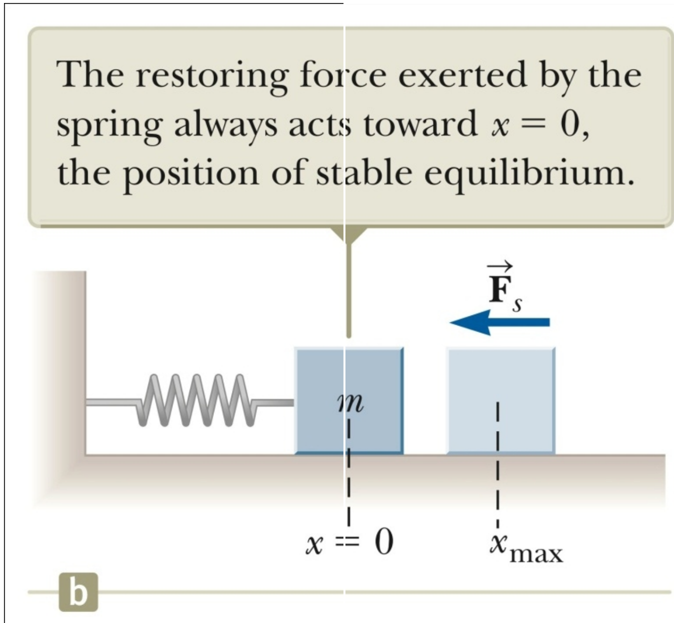
$\rightarrow f_k d = 334$ J

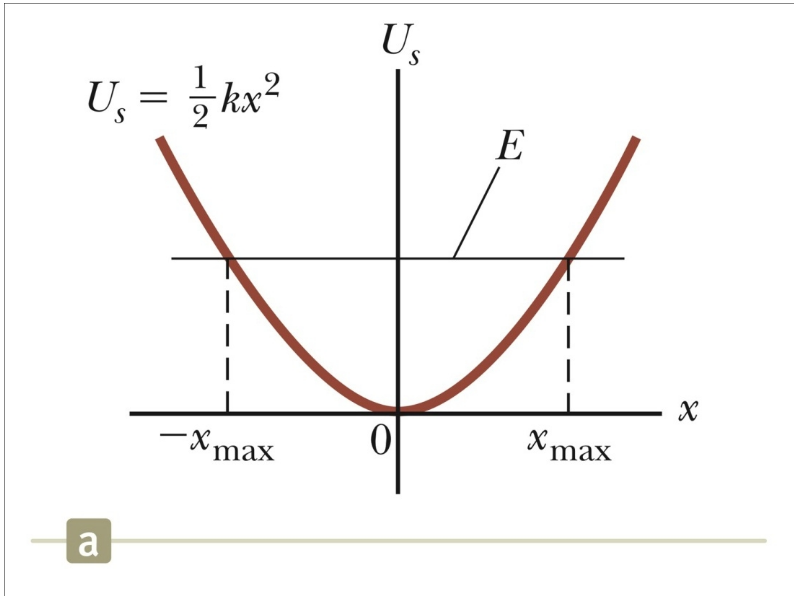
P. 29) Ski starts from rest and slides down 28° incline 85 m long. a) if μ_k=0.09, what is the ski's speed at base of the incline?

Soln: $\Delta K + \Delta U_g = -f_k d$ Find v_f buried in ΔK

vi=0, $mg\Delta y$= mg85sin28=mg(40m), d=85 m, f_k=0.09mgcos28

Stable and unstable equilibrium





Motion in a system can be observed in terms of a graph of its position and energy.

In a spring-mass system example, the block oscillates between the turning points, $x = \pm x_{max.}$

The block will always accelerate back toward $x = 0$.

Stable and unstable Equilibrium

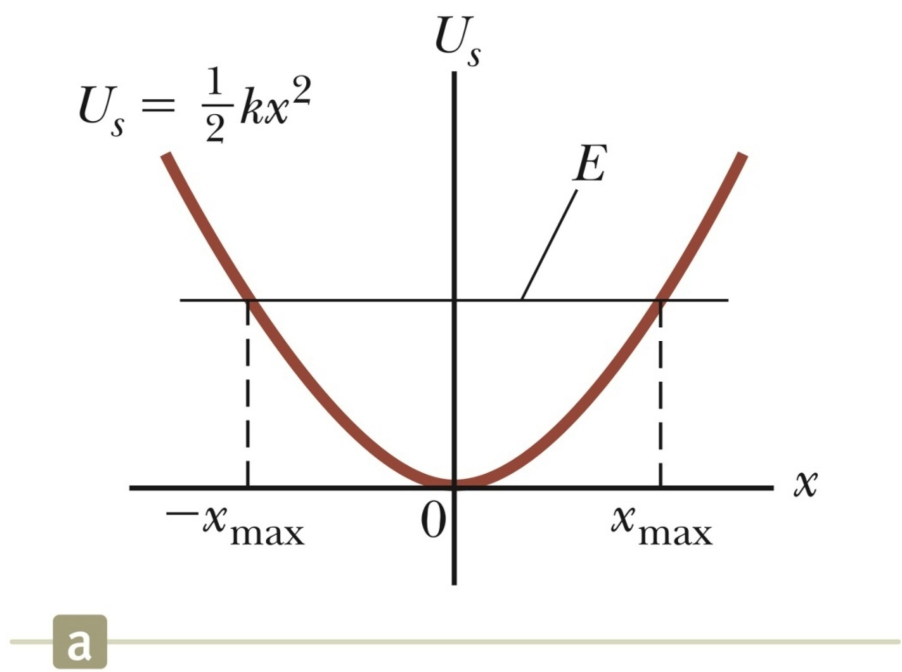
Equilibrium positions occur where the slope of the graph is 0. That's where the force is 0 because F_x=-dU/dx.

The x = 0 position is one of ***stable equilibrium.***

- Any movement away from this position results in a force directed back toward x = 0.

Configurations of stable equilibrium correspond to those for which $U(x)$ is a local minimum.

$x = x_{max}$ and $x = -x_{max}$ are called the turning points.



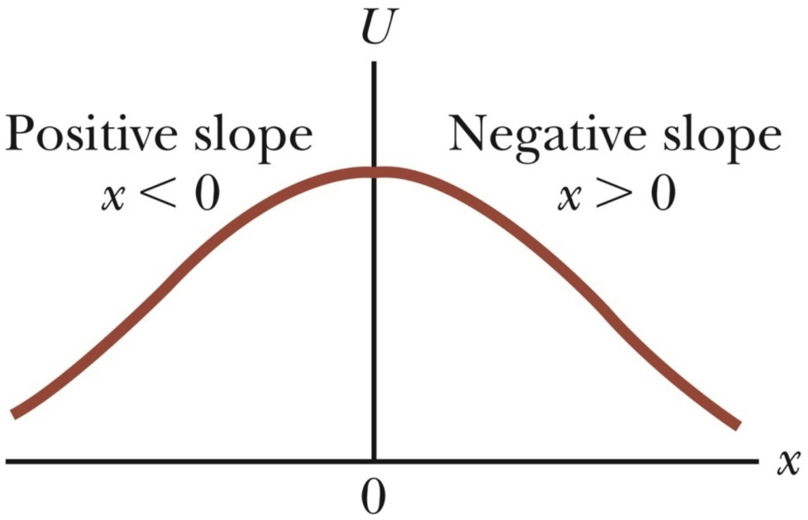
Energy Diagrams and Unstable Equilibrium

For this energy diagram, $F_x = 0$ at $x = 0$, so the particle is in equilibrium.

However, a slight displacement to the left or right leads to the particle moving away from x=0.

This is an example of ***unstable equilibrium.***

Configurations of unstable equilibrium correspond to those for which $U(x)$ is a local maximum.



Neutral Equilibrium

Neutral equilibrium occurs in a configuration when U is constant over some region.

A small displacement from a position in this region will produce neither restoring nor disrupting forces.

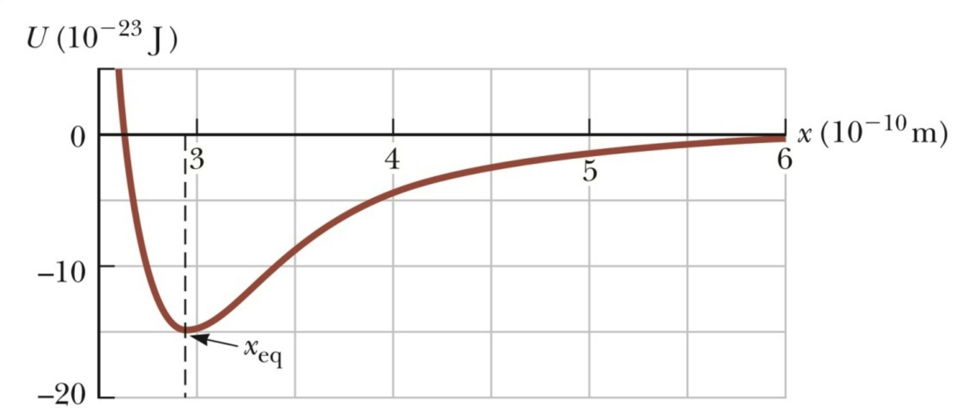
An example would be an energy diagram with an extended, flat region (where the slope is 0, but it is neither a maximum nor a minimum).

Ex) (P. 73) Graph 8-43

Potential Energy in Molecules

There is potential energy associated with the force between two neutral atoms in a molecule which can be modeled by the *Lennard-Jones* function.

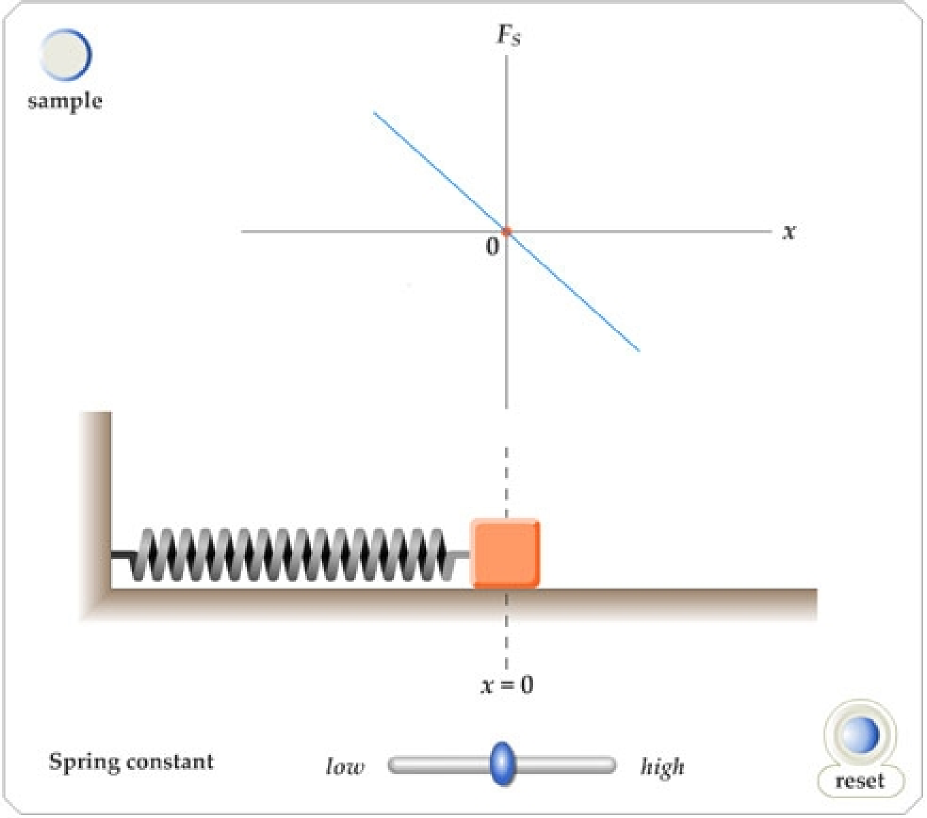
$$U(x) = 4\in\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$



Find the minimum of the function (take the derivative and set it equal to 0) to find the separation for stable equilibrium.

The graph of the Lennard-Jones function shows the most likely separation between the atoms in the molecule (at minimum energy).

7.9 Hooke's Law and Restoring Forces



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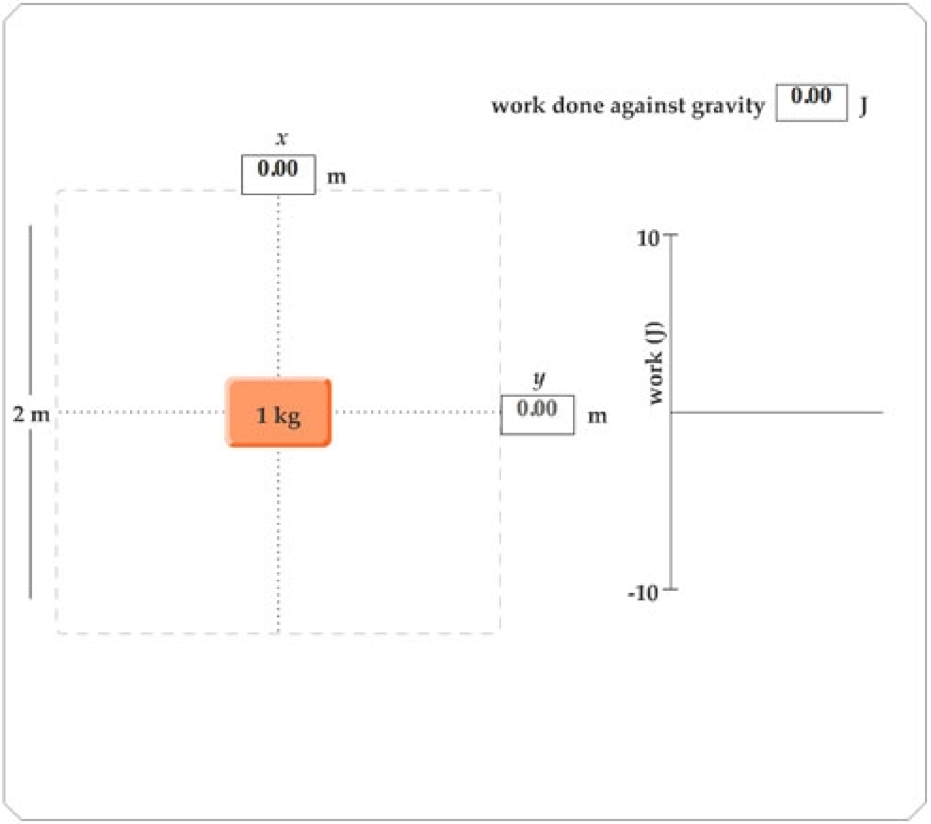
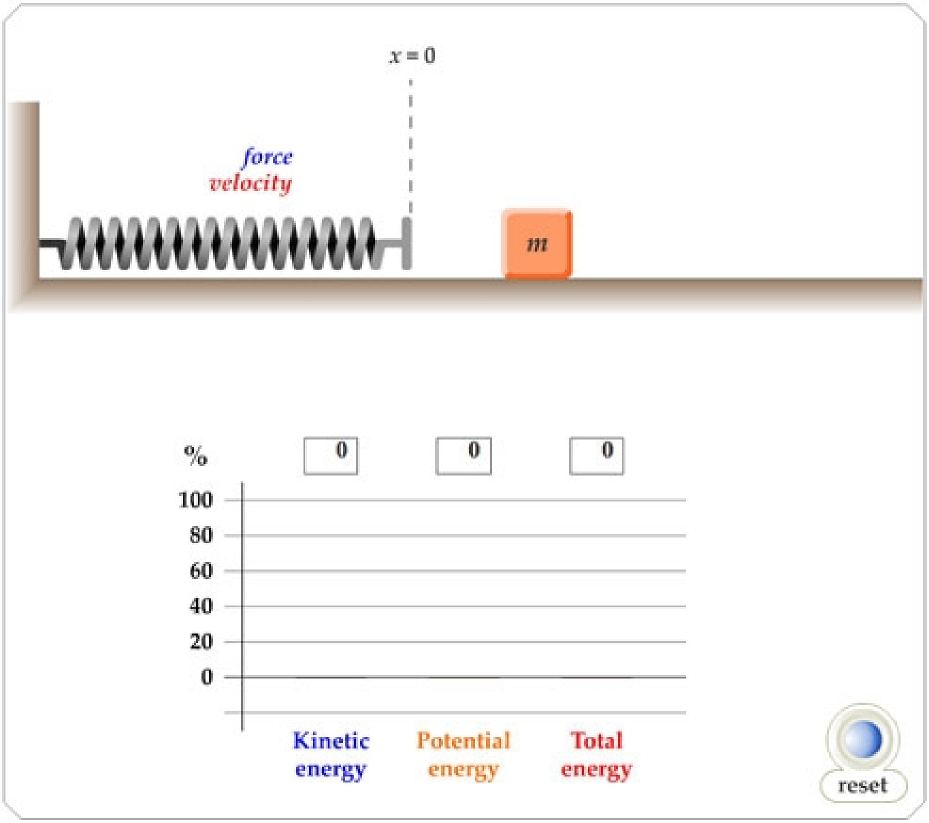
7.15 Work Done By External Agent on System of the Book



7.16 Conservation Mechanical Energy in Block-Spring System



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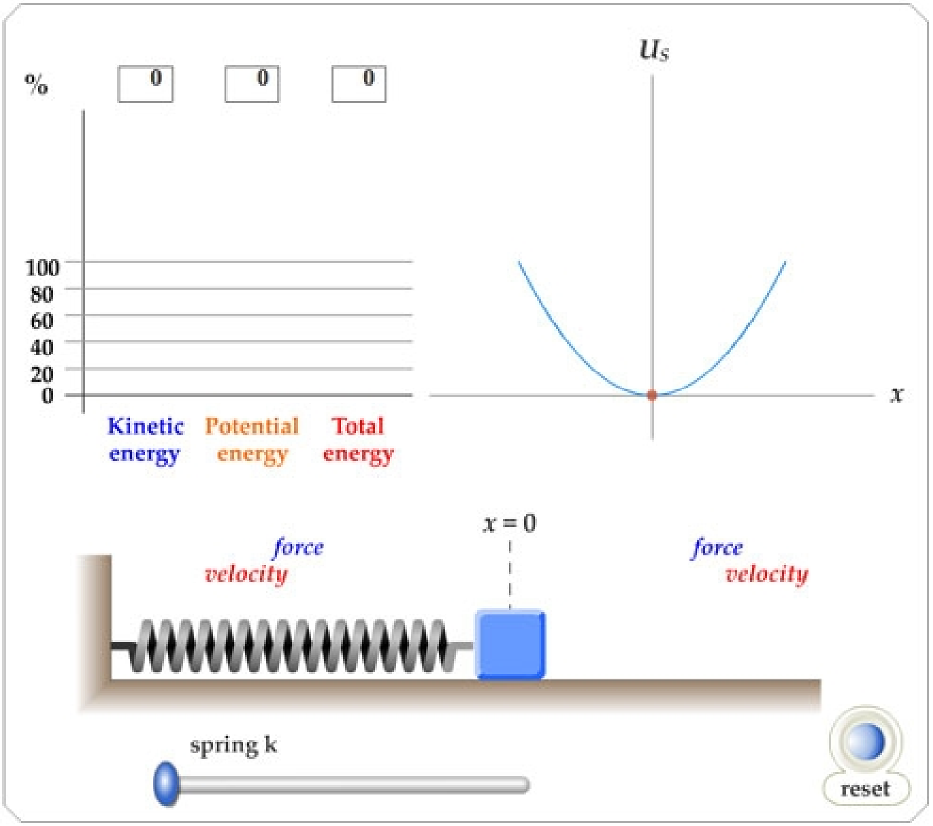
7.18 Work When Kinetic Friction is Present

7.20 An Oscillating Block-Spring System



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7.20b Conservation of Mechanical Energy for a Pendulum