Physics 2311. Homework 3 on Chs. 2-4

Ch. 2 P. 37,40,43,45, Ch. 3 P. 75, 79, Ch. 4 P. 18, 21, 26, 29, 31, 33, 38, 39

2.37 Write vectors in component form.

- $\vec{A} = 8.66\hat{\imath} + 5.00\hat{\jmath}$
- $\vec{B} = 3.0\hat{\imath} + 4.0\hat{\jmath}$
- $\vec{C} = 6.00\hat{\imath} - 10.39\hat{\jmath}$
- $\vec{D} = -16.0\hat{\imath} + 12.0\hat{\jmath}$
- $\vec{F} = -17.3\hat{\imath} - 10.0\hat{\jmath}$

2.40 Sledge pulled by horses on flat terrain.

- Express $\vec{F} = -2980.0\hat{\imath} + 8200.0\hat{\jmath}$ N in polar form.
- (R,θ) has R = $\sqrt{2980^2 + 8200^2}$ N = 8724.7 N
- $\theta = \arctan \frac{8200}{-2980} + 180 = 109.97°$.
- (R,θ) = **(8724.7m, 109.97°or 19.97°W of N)**.

2.43 Points $P_1(2.500m, \pi/6)$ and $P_2(3.800m, 2\pi/3$ have cartesian coordinates ...

- $\pi/6(180/\pi) = 30°$, $2\pi/3\frac{180}{\pi} = 120°$
- $P_{1x} = 2.5\cos 30 = 2.165$, $P_{1y} = 2.5\sin 30 = 1.250$
- $P_{2x} = 3.8\cos 120 = -1.90$, $P_{2y} = 3.8\sin 120 = 3.291$
- So P_1 = **(2.165,1.250)** and P_2 = **(-1.900,3.291)**
- Find Distance between P_1 and P_2:
- $\sqrt{(P_{2x} - P_{1x})^2 + (P_{2y} - P_{1y})^2}$= **4.549 m**
- NOTE: textbook sol'n manual wrong.

2.45 Find distance between A (2.00m, -4.00m) and B (-3.00m, 3.00m).

- $D = \sqrt{(-3-2)^2 + (3 - -4)^2}$ = **8.60 m**
- Convert to polar form:
- $R_A = \sqrt{2^2 + 4^2}$, and $\theta_A = \arctan(\frac{-4}{2})$, etc.
- $(R_A, \theta_A) = (4.47m, -63.43°)$ or $(4.47m, -0.352\pi)$
- $(R_B, \theta_B) = (4.243m, 135°)$ or $(3\sqrt{2}, 0.75\pi)$
- NOTE: textbook sol'n for vector A wrong.

3.75 Kangaroo can jump 2.5 m high.

a Find vertical component of velocity when leaving ground.

- Use $v_f^2 - v_i^2 = 2a_y(y_f - y_i)$
- $0 - v_i^2 = 2(-9.8)(2.5 - 0.0)$
- $v_i =$ **7.0 m/s**

b How long is it in the air?

- This is $2t_{max}$, where t_{max} is time to reach max height, or time when $v_y = 0$.
- $v_f = v_i + at$ so
- $0 = 7.0 - 9.8t_{max}$ gives $t_{max} = \frac{-7.0}{-9.8} = 0.714$ sec
- So $2t_{max}$ =**1.43 sec**.

3.79 Rocket's acceleration is $a(t) = A - Bt^{1/2}$

a What are the units of A and B given that $[a(t)] = L/T^2$?

- $[A] = L/T^2$, so units are m/s^2
- $[B][t^(1/2)] = L/T^2$, so $[B] = LT^{-2.5}$ and units are m/s^2.5

b If $v(t = 0) = v_i = 0$, "how does the velocity vary between $t = 0$ and $t = t_0$?

- NOTE: usually t_0 <u>is</u> the initial time. They should have used t_f.
- $\Delta v = \int a(t)dt = \int (A - Bt^{1/2})dt$
- $\Delta v = (At - \frac{2B}{3}t_f^{3/2} + C)|_0^{t_f}$
- $\Delta v = v(t_f) = At_f - \frac{2B}{3}t_f^{3/2}$

c If $x(0) = 0$, what is $x(t_f)$?

- $\Delta x = x(t_f) - x(0) = \int v(t)dt$
- $\Delta x = x(t_f) = \frac{A}{2}t^2 - \frac{2B}{3}t_f^{3/2} + C$
- $x(t_f) = \frac{A}{2}t_f^2 - \frac{4B}{15}t_f^{5/2}$

4.18 Particle's position changes from $\vec{r_1}$ to $\vec{r_2}$, find it's displacement

- $\Delta\vec{r} = \vec{r_2} - \vec{r_1} = (-4.0 - 2.0)\hat{i} + (3 - 3)\hat{j} = -6\hat{i} + 0\hat{j}$ cm

4.21 Cyclist's path has three legs: $\vec{A} = 5$ km E, $\vec{B} = 10$ km 20°W of N, $\vec{C} = 8$ km W. Find displacement.

- Sol'n: let East be $+\hat{i}$, W be $-\hat{i}$, N be $+\hat{j}$, etc.
- $\vec{A} = 5\hat{i}$, $\vec{B} = -3.42\hat{i} + 9.40\hat{j}$ km, $\vec{C} = -8\hat{i}$
- Displacement is $\Delta\vec{r} = -6.42\hat{i} + 9.4\hat{j}$ km or 11.38 km 34.3°W of N.

4.26 $\vec{r}(t) = 3t^2\hat{i} + 5.0\hat{j} - 6.0t\hat{k}$ m

a Find $\vec{v}(t) = \frac{d}{dt}\vec{r}$ and $\vec{a}(t) = \frac{d}{dt}\vec{v}$

- $\vec{v}(t) = 6t\hat{i} + 0\hat{j} - 6.0\hat{k}$ m/s
- $\vec{a}(t) = 6\hat{i} + 0\hat{j} + 0\hat{k}$ m/s^2

b Find $\vec{v}(0)$ and $\vec{a}(0)$.

- $\vec{v}(0) = 0\hat{i} + 0\hat{j} - 6.0\hat{k}$ m/s
- $\vec{a}(0) = 6.0\hat{i} + 0\hat{j} + 0\hat{k}$ m/s^2

4.29 For $t > 0$, $\vec{r}(t) = 3.0t^2\hat{i} - 7.0t^3\hat{j} - 5.0t^{-2}\hat{k}$

a Find $\vec{v}(t) = \frac{d}{dt}\vec{r}$

- $\vec{v}(t) = 6t\hat{i} - 21.0t^2\hat{j} + 10.0t^{-3}\hat{k}$ m/s

b Find $\vec{a}(t) = \frac{d}{dt}\vec{v}$

- $\vec{a}(t) = 6\hat{i} - 42.0\hat{j} - 30.0t^{-4}\hat{k}$ m/s^2

c Find $\vec{v}(2.0) = \frac{d}{dt}\vec{v}$

- $\vec{v}(2) = 12\hat{i} - 84.\hat{j} + 1.25\hat{k}$ m/s^2

d Find $v(1.0)$ and $v(3.0)$

- $v(1) = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{6^2 + 21^2 + 10^2}$
- $v(1) = 24.0$ m/s
- $v(3) = sqrt 18^2 + 189^2 + (10/27)^2 = 190$ m/s

e Find $\vec{v}_{avg}$ between $t = 1$ and 2 sec.

- $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$
- $\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{2-1}$
- $\vec{v}_{avg} = (3(2)^2\hat{i} - 7(20)^3\hat{j} - 5/4\hat{k}) - (3\hat{i} - 7\hat{j} - 5\hat{k}$
- $\vec{v}_{avg} = 9\hat{i} - 49\hat{j} + 3.75\hat{k}$ m/s

4.31 $\vec{r}(t) = \cos(1.0t)\hat{i} + \sin(1.0t)\hat{j} + t\hat{k}$

a Find $\vec{v}(t)$. $\vec{v}(t) = -\sin(1.0t)\hat{i} + \cos(1.0t)\hat{j} + \hat{k}$

b Find $\vec{a}(t)$. $\vec{a}(t) = -\cos(1.0t)\hat{i} - \sin(1.0t)\hat{j} + 0\hat{k}$

4.33 Bullet shot horizontally from $y_0 = 1.5$ m with speed $v_i = 200$ m/s.

a How much time elapses before bullet hits ground?

- $y(t_f) = y_0 + v_{0y}t_f + \frac{a_y}{2}t_f^2$
- $0 = 1.5 + 0 - 4.9t_f^2$
- Using $y_0 = 1.5$m, $v_{0y} = 0$, $a_y = -9.8$ m/s^2
- $-1.5 = -4.9t_f^2$, or $t_f = \sqrt{\frac{1.5}{4.9}}$=**0.55 sec**.

b How far does it go horizontally?

- $x(t_f) = x_0 + v_{0x}t_f + 0$
- $x(t_f) = 0 + 200(0.553s) =$ **110 m**

4.35 Dart thrown horizontally at 10 m/s straight at bullseye.

a How far does the dart drop below bullseye?

- $y(t_{hit}) = y_0 + v_{oy}t_{hit} - 4.9t_{hit}^2$
- Find t_{hit}: $t_{hit} = \frac{x_f}{v_{0x}}$=(2.4m)/(10m/s)=0.24 sec
- drop = y(0.24s)-y(0)=$-4.9(0.24)^2$=**-0.28 m**

b What does this tell you about how professional dart players throw? Ans: they throw a little high ($v_{0y} > 0$).

4.38 Pitcher throws at $v_0 = 40$ m/s horizontally

a How long does it take for ball to go 16.7 m?

- Use $\Delta t = \frac{d}{v_0}$
- $\Delta t = \frac{16.7}{40}$=0.42 sec

b How far does ball drop in this time?

- drop = $|y_f - y_i| = |v_{iy}t - \frac{9.8}{2}t^2|$
- drop = $|0 - 4.9(0.4175)^2| = 0.85$ m

4.39 Projectile shot at 30° elevation.

a Find initial speed

- Use $\Delta t = 2t_{max} = 20.$ seconds.
- Where $t_{max} = \frac{v_i \sin\theta}{g}$
- $v_i =$ **196 m/s.**

b Find maximum height

- Use $y_{max} = y(t_{max}) = y_0 + v_i t_{max} - 4.9t_{max}^2$
- with t_{max} = 10 sec, and v_i =196 m/s.
- y_{max} = 980-490 = **490 m**

c What is the range?

- Use $R = \frac{v_i^2 \sin 2\theta}{g}$
- $R =$ **3400 m**

d What is the displacement from t=0 to t=15 sec?

- Use $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
- with $\Delta\vec{r}_f = 196(15)\cos 30\hat{i} + [196(15)\sin 30 - 4.9(15s)^2]\hat{j}$
- $\Delta\vec{r} =$ **2550 m$\hat{i}$ + 368 m$\hat{j}$**