W9 D3

Hwk Ch.9 P. 2, 3, 4, 10, 11, 12, 16, 17, 20, 24, 26 } Due Mon.

- Read 9.1–9.6 • See online Examples for Ch.9.

Notes: Make use of tutoring & office hours

Ch. 8 Hwk key online.

Quiz 4 $\mu = 5.3/8 = 66.3\%$

Moodle updated

TODAY: Review Quiz 4

Linear Momentum (Ch.9)

Compare to KE

Conservation Law derived from Newton's 3rd

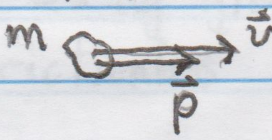
Ex

Types of Collisions

DEMO: Newton's Cradle

Linear Momentum

* $\vec{p} = m\vec{v}$
  - Vector
  - Units: $kg\,m/s$ (no special unit)



$p_x = mv_x$, $p_y = mv_y$, $p_z = mv_z$

* $K = \frac{1}{2}mv^2$ was a scalar (Kinetic energy)

$K \propto v^2$ $|\vec{p}| \propto v$

Ex) $m_1 = 4m_2$ and $K_1 = K_2$. What is $\frac{p_1}{p_2}$?

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_2 v_2^2$$

$$4m_2 v_1^2 = m_2 v_2^2$$

$$4v_1^2 = v_2^2$$

$$2 = v_2/v_1 \text{ or } \underbrace{\frac{v_1}{v_2} = \frac{1}{2}}$$

$$\therefore \frac{p_1}{p_2} = \frac{m_1 v_1}{m_2 v_2} = \frac{4m_2\,1}{m_2\,2} = \boxed{2}$$

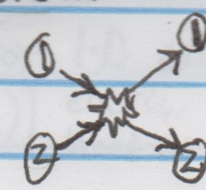
→ More massive object has smaller v

→ More massive object has larger p

Momentum (cont.)

Conservation of $\vec{p}$ from Newton's 3rd...

Consider 2 objects colliding:



At every instant...

$$\underbrace{\vec{F}_{21}}_{\text{by 2 on 1}} = -\vec{F}_{12}$$

So $m_1 \vec{a}_1 = -m_2 \vec{a}_2$ so $m_1\vec{a}_1 + m_2\vec{a}_2 = 0$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$\frac{d(m_1\vec{v}_1)}{dt} + \frac{d(m_2\vec{v}_2)}{dt} = 0$$

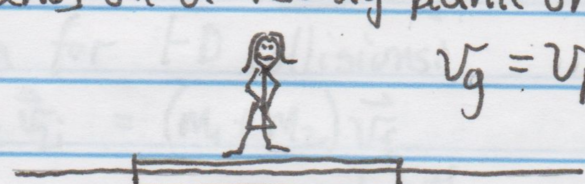
or $\frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2) = 0$

But $m_1\vec{v}_1 + m_2\vec{v}_2$ = the total momentum of the system

$\therefore \underbrace{\sum_{i=1}^{N} \vec{p}_i}_{\text{"The total linear momentum"}}$ is conserved in an isolated system

Example of 1-D conservation of momentum

Ex) A 45 kg girl stands on a 150 kg plank on a frictionless lake



$v_g = v_p = 0$

If the girl walks at $1.5\hat{\imath}$ m/s relative to plank,

relative to ground

a) What is v_{plank} rel. to lake and b) v_{girl} rel. to lake.

Sol'n $\vec{v}_g - \vec{v}_p = 1.5\hat{\imath}$ m/s $\rightarrow \vec{v}_g = 1.5\hat{\imath} + \vec{v}_p$

Cons. of $\vec{p}$: $\sum \vec{p}_{init} = \sum \vec{p}_{final}$

$$0 = m_g\vec{v}_g + m_p\vec{v}_p$$

$$0 = m_g(1.5\hat{\imath} + \vec{v}_p) + m_p\vec{v}_p$$

$$-(1.5\hat{\imath})m_g = (m_g + m_p)\vec{v}_p \Rightarrow \vec{v}_p = \frac{-1.5\hat{\imath}(45)}{(45+150)} = \boxed{-0.346 \text{ m/s } \hat{\imath}}$$

(b) $\vec{v}_g = 1.5\hat{\imath} - 0.346\hat{\imath}$

$$\boxed{\vec{v}_g = 1.154 \text{ m/s } \hat{\imath}}$$

Types of Collisions

1. Elastic ⇒ Kinetic energy is conserved.

   * special equations are derived for 1-D collisions (see text) (Sec. 9.4)

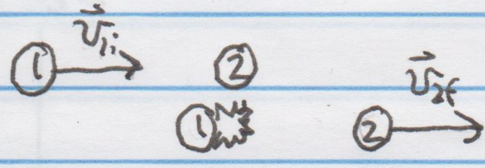
     (9.20) $\boxed{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})}$ elastic, head-on

     where $v_{1i} = |\vec{v}_{1i}|$, etc.

     (9.21) $$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

   * $$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

   Ex) Try $m_1 = m_2$ and $v_{2i} = 0$

   $$v_{1f} = 0 v_{1i} + 1 \cancelto{0}{v_{2i}} = 0$$
   $$v_{2f} = 1 v_{1i} + 0 v_{2i} = v_{1i}$$

2. Inelastic ⇒ Kinetic Energy is not conserved.
   * $KE_f < KE_i$
   * Momentum is still conserved!
   * Most real-life collisions

3. Perfectly inelastic ⇒ KE lost <u>and</u> objects stick together
   * Special equation for 1-D collisions:

     (9.14) $m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$

     both masses share $\vec{v}_f$

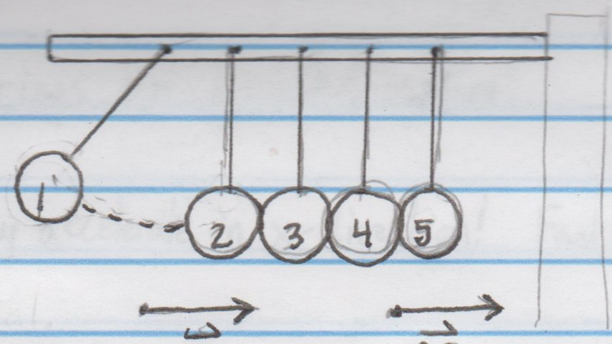
** All 3 types conserve momentum.

DEMO: "Newton's Cradle" or "Executive Time-Waster"

Predict what happens:

1) One ball released.

$m_1 = m_2 = m_3 = m_4 = m_5$

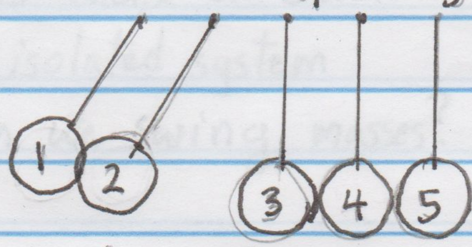


Just before collision, $\vec{v}_i$ is horizontal. $\vec{v}_{1i}$ $\vec{v}_{5f}$

Conservation of Momentum: $\sum \vec{p}_{init} = \sum \vec{p}_{final}$

$m_1 \vec{v}_i = m_5 \vec{v}_f$

2) Two balls released.



Cons. of Momentum: $(m_1 + m_2)\vec{v}_i = (m_4 + m_5)\vec{v}_f$

3) 3 balls released from left. ... from right.

4) 4 balls released from right.

RESULTS: we see symmetry!

energy gradually lost - not perpetual motion

5) Why can't m_4 & m_5 rise up at half the speed of m_1 in case #1?

$\xrightarrow{v_i}$ OOOOO

OOO OO→ v_f

$m v_i = (2m) v_f$

$\rightarrow v_f = \frac{1}{2} v_i$

$\vec{p}$ conserved

But what would K_i & K_f be? $K_i = \frac{1}{2} m v_i^2$, $K_f = \frac{1}{2}(2m)(\frac{1}{2}v_i)^2 = \frac{1}{4} m v_i^2$

6) What happens if 4 & 5 are stuck together?

Assume elastic collision & use Eq. 9.21 & 9.22. Here "m_2" $= m_4 + m_5 = 2m$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) \cancel{v_{2i}}$$

$$v_{1f} = \left(\frac{1-2}{3}\right) v_{1i} + 0$$

$$v_{1f} = -\frac{1}{3} v_{1i}$$

"blowback!"

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) \cancel{v_{2i}}$$

$$v_{2f} = \frac{2}{3} v_{1i} + 0$$

$$v_{2f} = \frac{2}{3} v_{1i}$$

DEMO (cont.)

7) Are the 5 ball bearings an isolated system?

→ $\vec{p}_{total}$ varies!

→ As masses descend, they accelerate in x-direction! $F_{net,x} \neq 0$

8) Placing the cradle on frictionless platform

→ Now table can't exert $F_{net,x}$ on cradle

→ Balls + cradle are isolated system

What happens when we swing masses?