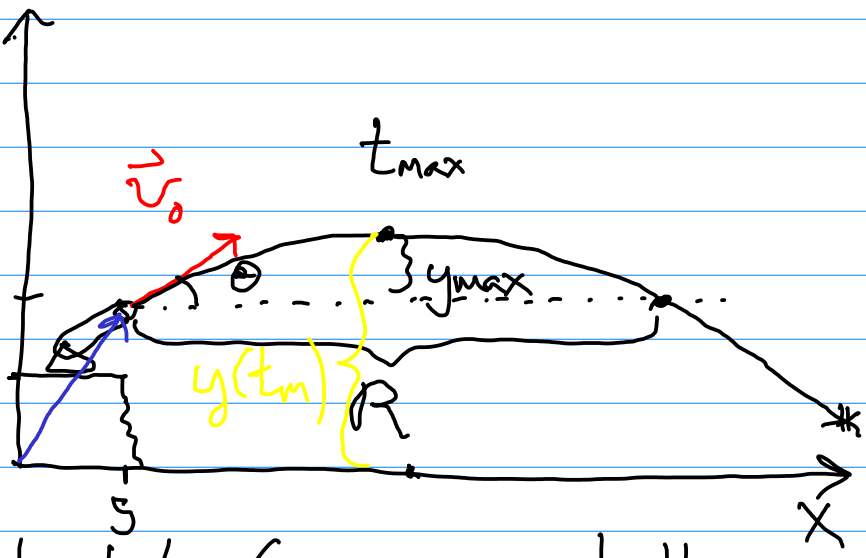
Derive t_{max}, y_{max}, and Range.

Ex) Cannonball

$\vec{r}_0 = 5m\hat{i} + 10m\hat{j}$

$\theta_i = 20°$

$|\vec{v}_0| = 30 m/s$



a) How long does it take for cannonball to rise to y_{max}?

Soln: (A) $v_y(t_{max}) = v_{y0} + a_y t_{max}$

$v_{y0} = v_0 \sin\theta_i$

$a_y = -9.8 m/s^2 = -g$

$v_y(t_{max}) = 0 = v_0 \sin\theta - g t_{max}$

Solve for $t_{max} = \frac{v_0 \sin\theta}{g}$ $\quad t_{max} = \frac{30 \sin 20°}{9.8}$

$= 1.05$ sec

b) How high does the cannonball get relative to the starting point?

Soln: Use Eqn. of Motion for $y(t)$

$y_{max} = y(t_{max}) - y_0$

$= \cancel{y_0} + v_{oy} t_{max} - \frac{1}{2} g t_m^2 - \cancel{y_0}$

$= v_0 \sin\theta_i t_{max} - \frac{1}{2} g t_m^2$

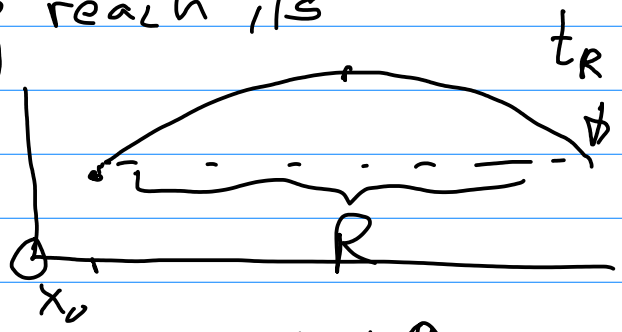
$y_{max} = v_0 \sin\theta \left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$

$$y_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

$$y_{max} = \frac{30^2 (\sin 20°)^2}{2(9.8)}$$

$$y_{max} = 5.37 \text{ m}$$

c) What is the range (ie. how far does it move horizontally to reach its starting ~~point~~ height.)



$$R = x(t_R) - x_0$$
$$= x_0 + v_{ox} t_R - x_0$$
$$= v_0 \cos\theta \, (2 t_{max})$$
$$= v_0 \cos\theta \left(2 \frac{v_0 \sin\theta}{g}\right)$$

$$t_{max} = \frac{v_0 \sin\theta}{g}$$

Trig ident.
$2 \sin\theta \cos\theta = \sin 2\theta$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$R = \frac{30^2 \sin(2 \times 20°)}{9.8} = 59. \text{m}$$

↳ Which θ gives R_{max}?

$\theta = 45°$

$\theta = 30 \rightarrow \theta = 60°$
same R!